*2011 International Conference on Circuits, System and Simulation*
*Proc .of CSIT vol.7 (2011) © (2011) IACSIT Press, Singapore*

# Near Theoretical Maximum Throughput Limits of CSMA/CA RTS CTS Protocol in IEEE 802.11 Wireless Networks Using Active Node Back-off Algorithm

Jesada Sartthong+ and Suvepon Sittichivapak

Department of Telecommunications Engineering, Faculty of Engineering
King Mongkut's Institute of Technology Ladkrabang, Bangkok, Thailand

**Abstract.** In this paper, we propose a new calculated contention window technique in back-off algorithm mode and its name is Active Node Back-off Algorithm (ANBA). The ANBA is produced for improved performance of CSMA/CA RTS CTS protocol for wireless local area network in IEEE802.11a/b/g standards. The performance of ANBA is compared with old back-off algorithms which are Binary Exponential back-off, G. Bianchi's back-off, S. Won Kang's back-off and the theoretical maximum throughput. The performance of ANBA is measured in terms of data link layer rate throughput parameters. Our numerical results indicate that the performance of ANBA is better than old back-off algorithm of which we respected. An analytical model based on Markov chain is introduced to compute the enhanced throughput of Distributed Coordination Function in IEEE 802.11a/b/g wireless local area network standards.

**Keywords:** CSMA/CA RTS CTS, Active Node Back-off Algorithm, IEEE802.11a/b/g.

## 1. Introduction

Today, wireless LAN networks IEEE 802.11 medium access control protocol is based on a carrier sensing multiple accesses with collision avoidance (CSMA/CA) technique and employed a binary exponential back-off (BEB) in Distribute Coordination Function (DCF) mode. In CSMA/CA technique, there are two access methods that are used under DCF, namely the basic access method and request-to-send (RTS) clear-to-send (CTS) access method. RTS/CTS technique has been introduced to reduce the performance degradation due to hidden terminal. In 2000, Giuseppe Bianchi developed a simple mathematical model for the IEEE802.11 DCF throughput performance analysis in Fig.1 [4] [5].We extended Bianchi's model to calculate throughput efficiency under the assumption of ideal channel conditions. Note that each node (station) is allowed to transmit only when its back-off timer reaches zero and at the beginning of each slot time.

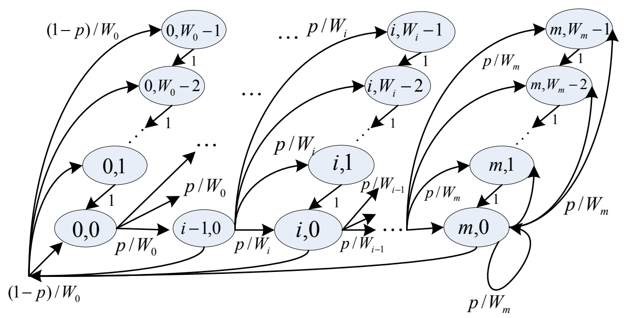



Fig. 1: Discrete Markov Chain Model (Bianchi's Model)

+ Corresponding author. Tel.: + 66 86-5454007.
*E-mail address*: sartthong@yahoo.com.

From Fig.1, the binary exponential back-off mechanism employed in DCF and the probability of collision is constant and independent for each transmitted data packets under saturation condition. The back-off process is represented by Discrete Markov chain model which is a stochastic process of two dimensions $\{s(t), b(t)\}$. The $s(t)$ is back-off stages due to collisions at time $t$ and $b(t)$ is the size of the contention window of a back-off entity at time $t$. The size of the contention window per back-off stage $i$ is calculated to: $W_i = 2^i W_{\min} \rightarrow 0 \le i < m$ and $W_{\max} = 2^m W_{\min} \rightarrow i \ge m$ . The $W_i$ is the contention windows at back-off stage $i$ and $m$ is the maximum back-off stage where $m = \log_2(W_{\max}/W_{\min})$.The $W_{\min}$ is minimum contention window and $W_{\max}$ is maximum contention windows. Bianchi's model assumes by an unlimited number of retransmission and that collision probability $p$ is independent from state $s(t)$ of the station. The state of each station is described by $(i,k)$ where $i$ indicates the back-off stage and takes the values $(0,1,2\ldots m)$ and $k$ indicates the back-off delay counter and takes the values $(0,1,2\ldots W_i - 1)$ in slot times. When a previous frame is sent successfully, a new frame arrives immediately and selects a back-off value $k_0$ uniformly in the integer rang $(0, W_0 - 1)$. The transition probabilities of the time discrete Markov model are defined as:

$$P = \{i,k|i,k+1\} = 1 \ , \quad 0 \le i \le m, 0 \le k \le W_i - 2 \ \text{ and } \ P = \{0,k|i,0\} = (1-p)/W_0, \ \ 0 \le i \le m, 0 \le k \le W_0 - 1 \quad (1)$$

$$P = \{i,k|i-1,0\} = p/W_i, \ \ 1 \le i \le m, 0 \le k \le W_i - 1 \ \text{ and } \ P = \{m,k|m,0\} = p/W_m, \quad i = m, 0 \le k \le W_m - 1 \quad (2)$$

Changing transition stages are referred by: $P = \{i_1, k_1 | i_0, k_0\} = P\{s(t+1) = i_1, b(t+1) = k_1 | s(t) = i_0, b(t) = k_0\}$

We let $\tau$ be the probability that a station (node) transmits in a generic back-off slot time at steady state. The value of $\tau$ can be calculated via equations as follows:

$$\tau = \sum_{i=0}^{m} b_{i,0} = b_{0,0} + b_{1,0} + b_{2,0} + \ldots b_{m,0}, \ b_{1,0} = p.b_{0,0} \rightarrow b_{2,0} = p.b_{1,0} = p^2.b_{0,0} \rightarrow \cdots b_{i,0} = p^i.b_{0,0}, \ \ b_{i,0} = p.b_{i-1,0}, \ \rightarrow 0 < i \le m \quad (3)$$

The values of $b_{i,k}$ are given by: $b_{i,k} = \lim_{t \to \infty} P\{s(t) = i, b(t) = k\} \ \ \rightarrow k \in (0,1,2\ldots W_i - 1), \ i \in (0,1,2\ldots m)$

The closed-form solution for Markov chain model has followed solution:

$$b_{m-1,0}.p = (1-p) b_{m,0} \quad \rightarrow b_{m,0} = \frac{p^m}{1-p} b_{0,0} \quad \rightarrow b_{i,k} = \frac{W_i - k}{W_i} b_{i,0} \quad \rightarrow b_{i,k} = \frac{W_i - k}{W_i} . \begin{cases} (1-p)\sum_{j=0}^{m} b_{j,0} & i = 0 \\ p.b_{i-1,0} & 0 < i < m \\ p.(b_{m-1,0} + b_{m,0}) & i = m \end{cases} \quad (4)$$

By means of relation (3), and making use of the fact that $\sum_{i=0}^{m} b_{i,0} = \frac{b_{0,0}}{(1-p)}$; expression all $b_{i,k}$ values are followed:

Equation (4) becomes: $b_{i,k} = \frac{W_i - k}{W_i} b_{i,0} \rightarrow i \in (0,m), k \in (0, W_i - 1)$

$$1 = \sum_{i=0}^{m} \sum_{k=0}^{W_i - 1} b_{i,k} \ \rightarrow 1 = \sum_{i=0}^{m} b_{i,0} \sum_{k=0}^{W_i - 1} \frac{W_i - k}{W_i} \quad \rightarrow 1 = \sum_{i=0}^{m} b_{i,0} \frac{2^i W_{\min} + 1}{2}$$

$$1 = \left[ \sum_{i=0}^{m-1} p^i b_{0,0} + \frac{p^m}{1-p} b_{0,0} \right] \left( \frac{2^i W + 1}{2} \right) \quad \rightarrow 1 = \frac{b_{0,0}}{2} \left[ W \sum_{i=0}^{m-1} (2p)^i + \frac{W 2^i p^m}{1-p} + \sum_{i=0}^{m-1} p^i + \frac{p^m}{1-p} \right] \quad (5)$$

We simplified $\frac{W 2^i p^m}{1-p} \cong \frac{W 2^m p^m}{1-p} \cong \frac{W(2p)^m}{1-p} \quad \rightarrow i = m$ using equation (5), we have

$$1 = \frac{b_{0,0}}{2} \left[ W \sum_{i=0}^{m-1} (2p)^i + \frac{W 2^m p^m}{1-p} + \sum_{i=0}^{m-1} p^i + \frac{p^m}{1-p} \right] \quad \rightarrow \frac{2}{b_{0,0}} = W \sum_{i=0}^{m-1} (2p)^i + \frac{W(2p)^m}{1-p} + \sum_{i=0}^{m-1} p^i + \frac{p^m}{1-p} \quad (6)$$

From $\sum_{i=0}^{m-1} (2p)^i = \frac{1 - (2p)^{(m-1)+1}}{1 - 2p} = \frac{1 - (2p)^m}{1 - 2p}$ and $\sum_{i=0}^{m-1} p^i = \frac{1 - p^m}{1 - p}$ using equation (6), we have

$$\frac{2}{b_{0,0}} = W \left[ \frac{1 - (2p)^m}{(1 - 2p)} \right] + \frac{W(2p)^m}{1-p} + \frac{1 - p^m}{1-p} + \frac{p^m}{1-p} \quad \text{Finally, we have} \quad b_{0,0} = \frac{2(1-2p)(1-p)}{(1-2p)(W+1) + pW(1-(2p)^m)} \quad (7)$$

The probability $\tau$ that a back-off entity is attempting a transmission in a generic slot is given through the summation over all probabilities $b_{i,0}$, that the back-off counter reaches zero:

$$\tau = \sum_{i=0}^{m} b_{i,0} = \frac{b_{0,0}}{1-p} = \frac{2(1-2p)}{(1-2p)(W+1) + pW(1-(2p)^m)} \quad (8)$$

In [5], the system throughput is the fraction of expected successful length over the expected cycle length, as following equation:

$$\mathit{Throughput} = \frac{P_S P_{tr} E[P]}{(1-P_{tr})\sigma + P_{tr} P_S T_S + P_{tr}(1-P_S) T_C} \text{, where } P_{tr} = 1 - (1-\tau)^n \text{ and } P_S = \frac{n\tau(1-\tau)^{n-1}}{P_{tr}} = \frac{n\tau(1-\tau)^{n-1}}{1-(1-\tau)^n} \quad (9)$$

Where $P_{tr}$ = Probability that in a slot time there is at least one transmission

$P_s$ = Probability that in a slot time there is successful

$E[P]$ = Average packet payload size in a slot time

$P_{tr}P_S E[P]$ = Average amount of Payload information successfully transmitted in a slot time

$P_{tr}P_S$ = Probability of successful transmission

$(1-P_{tr})$ = Probability that slot time is empty

$P_{tr}(1-P_S)$ = Probability of collision transmission

$T_C$ = Collision transmission time

$T_S$ = Successful transmission time, $\sigma$ = Empty slot time and $n$ = Number of active nodes stations

As in [5], G. Bianchi who derived optimal contention window is given by: $W_{optimal} \approx n\sqrt{2T_S}$ and as in [6], S. Won Kang who derived optimal contention window is given by: $W_{optimal} \approx n$ .

## 2. Active Node Back-off Algorithm (ANBA)

We suggested a mechanism to modify the contention window size based on the number of active stations by used the maximum function theory. From the throughput equation (9), the optimum value of W (contention window) is obtained by the solution of the equation:

$$\frac{d(Throughput)}{dW} = \frac{d}{dW}\left[\frac{P_S P_{tr} E[P]}{(1-P_{tr})\sigma + P_{tr}P_S T_S + P_{tr}(1-P_S)T_C}\right] = 0 \tag{10}$$

We consider only the relation between the throughput and contention window parameters; therefore, we let other parameters which are constant; therefore, we assigned $E=(1-2p)$, $F=p(1-2^m p^m)$, $H=E+F$ , $I=\sigma$, $J=E[P]$, $K=T_S$ and $L=T_C$ . The derivative throughput equation can be rewritten by:

$$\frac{d}{dW}\left[\frac{2nJE(-E+WH)^{n-1}}{L(E+WH)^n+(I-L)(-E+WH)^n+(2nKE-2nEL)(-E+WH)^{n-1}}\right]=0 \tag{11}$$

In (11), we let $Q=2nJE$, $O=(I-L)$ and $Y=(2nKE-2nKL)$ so that:

$$\frac{d}{dW}\left[\frac{Q(-E+WH)^{n-1}}{L(E+WH)^n+O(-E+WH)^n+Y(-E+WH)^{n-1}}\right]=0$$

$$\vdots$$

$$(-E+WH)^2=(-E)^2+2(-E)(WH)+(WH)^2=0$$

$$H^2W^2-2EHW+E^2=0$$

From $ax^2+bx+c=0 \rightarrow x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ , where $a=H^2$, $b=-2EH$ and $c=E^2$ , we have:

$$W=\frac{-(-2EH)-\sqrt{(-2EH)^2-4H^2E^2}}{2H^2} \tag{12}$$

We return $E$, $F$ and $H$ into (12).Finally, the optimal contention window which depended on active nodes in back-off mode is given by:

$$W_{ANBA}=\frac{2(1-2p)[(1-2p)+p(1-2^m p^m)]-\sqrt{\begin{bmatrix}4(1-2p)^2((1-2p)+p(1-2^m p^m))^2-\\4((1-2p)+p(1-2^m p^m))^2(1-2p)^2\end{bmatrix}}}{2((1-2p)+p(1-2^m p^m))^2}\times n \tag{13}$$

As equation (13), it is named Active Node Back-off Algorithm (ANBA).

## 3. The Theoretical Maximum Throughput

The theoretical maximum throughput is defined as the ratio of the spent time in transmission of successfully received packets to the maximum time available for transmission. Figure 2 illustrates the data frame exchanged sequence of CSMA/CA protocol in case of RTS CTS mechanism for IEEE802.11a/b/g standards. The MAC Service Data Unit (MSDU) size includes all the overheads (header, trailer) of above layers and treats as the payload at MAC layer for our calculations. [1][2] and [3]

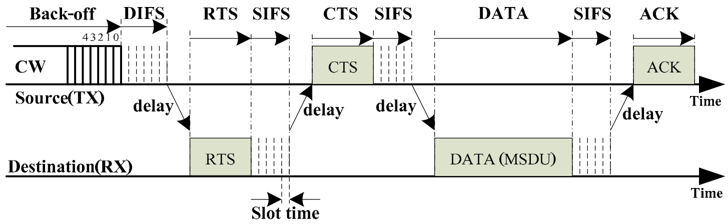



Fig. 2: Timing of the CSMA/CA RTS CTS Protocol Access Technique

The theoretical maximum throughput of CSMA/CA RTS CTS protocol in IEEE802.11a/b/g standard is given by:

$$Theory\ throughput = \frac{Transmitted\ Data(MSDU)}{Transmission\ Cycle\ Duration} = \frac{8 \times MSDU(size)}{T_{S[CSMA/CA\ RTS\ CTS]}}(bps) \tag{14}$$

Where

$$T_{S[CSMA/CA\ RTS\ CTS]} = T_{RTS} + 3T_{SIFS} + 4T_{delay} + T_{CTS} + T_{MSDU(size)} + T_{ACK} + T_{DIFS} + T_{Backoff}$$

$$T_{C[CSMA/CA\ RTS\ CTS]} = T_{Backoff} + T_{DIFS} + T_{RTS} + T_{delay}$$

We can find the transmission times for $T_{RTS}$, $T_{CTS}$, $T_{MSDU}$ and $T_{ACK}$ as follows:

$$T_{RTS\ 802.11\ a/g} = T_{\Pr eamble} + T_{Signal} + T_{SYM}\left(\frac{L_{Service} + L_{Tail} + 8L_{RTS}}{N_{DBPS}}\right) \tag{15}$$

$$T_{CTS\ 802.11\ a/g} = T_{ACK\ 802.11\ a/g} = T_{\Pr eamble} + T_{Signal} + T_{SYM}\left(\frac{L_{Service} + L_{Tail} + 8L_{ACK}}{N_{DBPS}}\right) + T_{EX} \tag{16}$$

$$T_{MSDU\ 802.11\ a/g} = T_{\Pr eamble} + T_{Signal} + T_{SYM}\left(\frac{L_{Service} + L_{Tail} + 8(L_{MACheader} + MSDU)}{Data\ rate}\right) + T_{EX} \tag{17}$$

$$T_{RTS\ 802.11b} = T_{\Pr eamble} + T_{PLCPheader} + \frac{8L_{RTS}}{RTS\ rate} \quad \text{and} \quad T_{CTS\ 802.11b} = T_{ACK\ 802.11b} = T_{\Pr eamble} + T_{PLCPheader} + \frac{8L_{ACK}}{ACK\ rate} \tag{18}$$

$$T_{MSDU\ 802.11b} = T_{\Pr eamble} + T_{PLCPheader} + \frac{8(L_{MACheader} + MSDU)}{Data\ rate} \tag{19}$$

$$T_{delay} \approx 1\mu s \rightarrow \frac{(Tx\ to\ Rx\ distance \approx 300m)}{(Radio\ waves\ propagation\ speed \approx 3\times10^{8}\ m/s)} \quad \text{, and} \quad T_{Backoff\ (Average)} = \frac{W_{\min} \times T_{Slot}}{2} \tag{20}$$

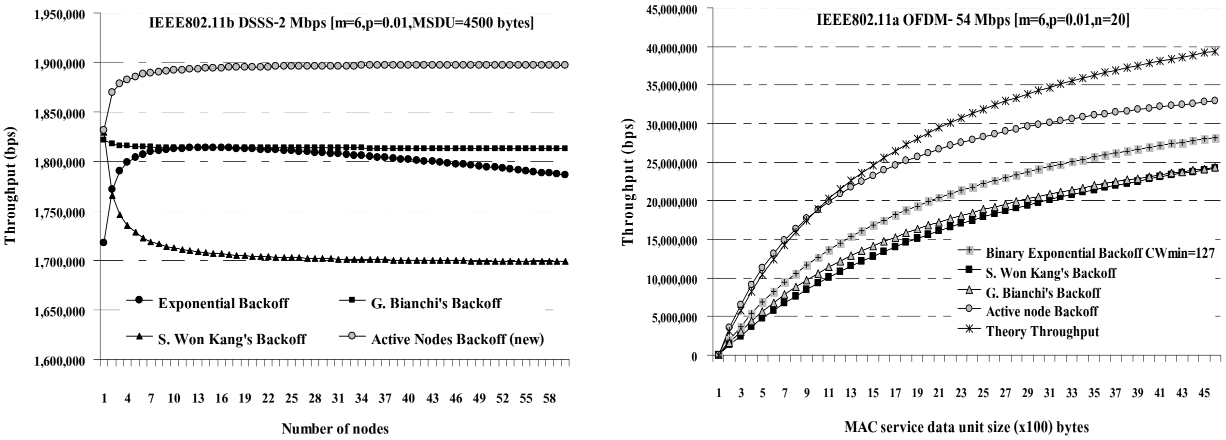
The Number of Data Bits per OFDM Symbol ($N_{DBPS}$) is 24, 48, 96 and 216 for OFDM-6 Mbps, OFDM-12 Mbps, OFDM-24 Mbps and OFDM-54 Mbps, respectively [1] [2] and [3]. The physical characteristics and parameter values of the investigated system of CSMA/CA RTS CTS protocol for IEEE802.11a/b/g are summarized in Table 1.

TABLE 1. The CSMA/CA RTS CTS Protocol Parameters in IEEE802.11a/b/g Standards

| Parameters | 802.11a | 802.11b | 802.11g | Parameters | 802.11a | 802.11b | 802.11g |
|---|---|---|---|---|---|---|---|
| $T_{SIFS}$ | 16 $\mu$S | 10 $\mu$S | 10 $\mu$S | $T_{EX}$ | - | - | 6 $\mu$S |
| $T_{DIFS}$ | 34 $\mu$S | 50 $\mu$S | 28 $\mu$S | $L_{RTS}$ | 20 bytes | 20 bytes | 20 bytes |
| $T_{SLOT}$ | 9 $\mu$S | 20 $\mu$S | 9 $\mu$S | $L_{MAC\ header}$ | 34 bytes | 34 bytes | 34 bytes |
| $T_{PLCP\ header}$ | - | 48 $\mu$S | - | $L_{Service}$ | 16 bits | - | 16 bits |
| $T_{\Pr eamble}$ | 16 $\mu$S | 144 $\mu$S | 16 $\mu$S | $L_{Tail}$ | 6 bits | - | 6 bits |
| $T_{SYM}$ | 4 $\mu$S | - | 4 $\mu$S | $L_{ACK}$ | 14 bytes | 14 bytes | 14 bytes |
| $T_{Signal}$ | 4 $\mu$S | - | 4 $\mu$S | $L_{CTS}$ | 14 bytes | 14 bytes | 14 bytes |

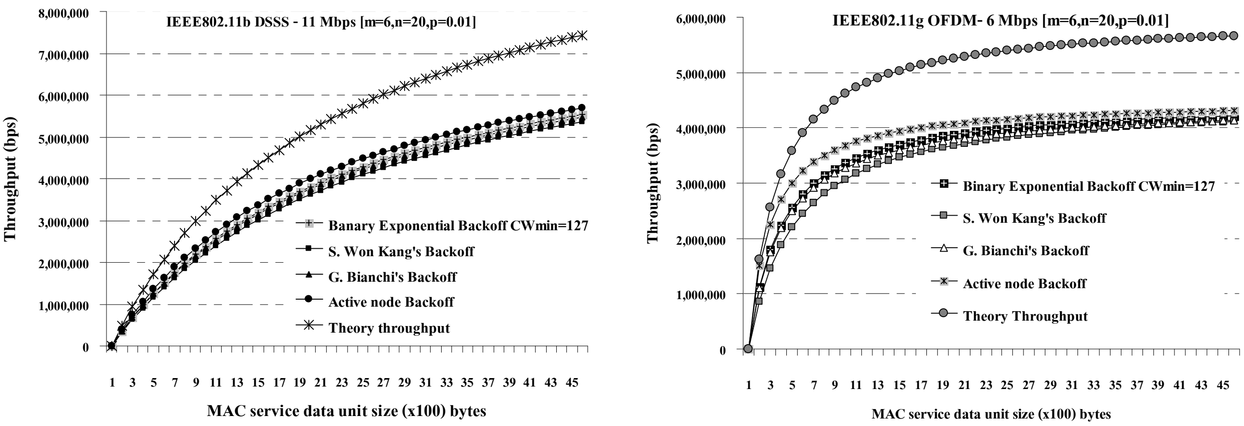
## 4. Numerical Results and Discussions

In order to validate our analytical models, we used MATLAB and MATCAD engineering calculation tools. In Fig. 3(a), we plot throughput efficiency that it depends on active nodes of the RTS/CTS scheme for a data rate of 2 Mbps in IEEE802.11b standard. The MAC service data unit size is fixed of MSDU=4500 bytes and the collision probability is fixed of $p$ = 0.01 and the number of back-off stages are fixed of m = 6. This figure compares throughput performance for the four back-off algorithms as a function number of nodes. From the picture, the throughput performance of Active Nodes Back-off Algorithm is higher than binary exponential back-off, Bianchi's back-off and Won Kang's back-off all cases when the numbers of nodes are increased in CSMA/CA RTS CTS access mechanism. In Fig. 3(b), 4(a) and 4(b) show the throughput efficiency of Active Node Back-off Algorithm that its throughput performance is compared with Binary Exponential back-off, G. Bianchi's back-off, S. Won Kang's back-off and the theoretical maximum throughput. When the MAC service data unit sizes are changed, the throughput of ANBA is higher than other back-off algorithm but less than the theoretical maximum throughput.




(a) (b)

Figure 3. (a) The throughput efficiency of ANBA in IEEE 802.11b and (b) in IEEE 802.11a Standard




(a) (b)

Figure 4. (a) The throughput efficiency of ANBA in IEEE 802.11b and (b) in IEEE 802.11g Standard

# 5. Conclusions

We introduced a new back-off algorithm of CSMA/CA RTS CTS protocol for IEEE802.11a/b/g wireless local area network standards; it is called Active Node Back-off Algorithm (ANBA). We study the throughput efficiency of each back-off algorithms in contention windows calculation mode. The numerical results showed that the performance of ANBA is better than old back-off algorithm when the number of nodes and MSDU sizes are changed. As future work, we plan to investigate the performance of ANBA in non-saturated condition network.

# 6. References

[1] IEEE Std. 802.11a Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY Specification: High-Speed Physical Layer Extension in the 5 GHz Band, IEEE802.11a WG September 1999.

[2] IEEE Std. 802.11b Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specification: High-Speed Physical Layer Extension in the 2.4 GHz Band, IEEE802.11a WG, September1999.

[3] IEEE Std. 802.11g Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specification: Amendment 4 Further Higher Data Rate Extension in the 2.4 GHz Band, IEEE802.11a WG, June

2003.

[4] G. Bianchi. Performance Analysis of the IEEE 802.11 Distributed Coordination Function. *in IEEE JSAC 2000*, **18(**3):535-547.

[5] G. Bianchi, L. Fratta and M. Oliveri. Performance Evaluation and Enhancement of the CSMA/CA MAC Protoco for 802.11 Wireless LANs. *in PIMRC*, October 1996.

[6] S. Won Kang. A Novel Estimation-based Back-off Algorithm in the IEEE 802.11 based Wireless Network. *IEEE CCNC*, October 2010.