MATH ④ ①

~~ชื่อ~~

เฉลย ปราบทอง

① $y'' - 10y' + 25y = 30x + 3$ ————

$(m^2 - 10m + 25)y = 0$

$(m^2 - 10m + 25) = 0$

$(m-5)(m-5) = 0$

$m_{1,2} = +5$

$Y_c = c_1 e^{5x} + c_2 x e^{5x}$ ————

หา Y_p จาก $g(x) = 30x + 3$ ———— ①

$Y_p = Ax + B$

$y' = \frac{d}{dx}(Ax + B) = A$ ———— ②

$y'' = \frac{dA}{dx} = 0$ ———— ③

แทน ② ③ ลงในโจทย์

$(0) - 10(A) + 25(Ax + B) = 30x + 3$

$-10A + 25Ax + 25B = 30x + 3$

$$25A = 30$$

$$A = \frac{30}{25} = \frac{6}{5}$$

$$-10A + 25B = 3$$

$$-10\left(\frac{6}{5}\right) + 25B = 3$$

$$-12 + 25B = 3$$

$$B = \frac{3+12}{25} = \frac{15}{25} = \frac{3}{5}$$

$$y_p = \frac{6}{5}x + \frac{3}{5}$$

$$y = y_c + y_p = C_1 e^{5x} + C_2 x e^{5x} + \frac{6}{5}x + \frac{3}{5}$$

[illegible] $y = c_1 e^{5x} + c_2 x e^{5x} + \frac{6x}{5} + \frac{6}{5}$

(2) (8)

Answer $y = e^{2x}(C_1 \cos 4x + C_2 \sin 4x) + 5x^2 + 4x + \frac{11}{10} + (-2x - \frac{12}{13}) e^{x}$

$$y'' - 8y' + 20y = 100x^2 - 26xe^{x}$$

$$(m^2 - 8m + 20) = 0$$

$$\cancel{(m+2)(m-10)}$$

$$\cancel{m_1 = -2}$$

$$\cancel{m_2 = 10}$$

$$\cancel{y_c = C_1 e^{-2x} + C_2 e^{10x}}$$

$$m_1, m_2 = \frac{-b \pm \sqrt{b^2 - 4Ac}}{2A}$$

$$m_1, m_2 = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(20)}}{2(1)}$$

$$m_1, m_2 = \frac{8 \pm \sqrt{64 - 80}}{2} = \frac{8 \pm \sqrt{-16}}{2}$$

$$m_1, m_2 = \frac{8 \pm \sqrt{16 \times (-1)}}{2} = \frac{8 \pm j4}{2}$$

$$m_1, m_2 = 4 \pm j2 = \alpha \pm j\beta$$

$$m_1 = \alpha + j\beta = 4 + j2$$

$$m_2 = \alpha - j\beta = 4 - j2$$

$\alpha = 4 \;,\quad \beta = 2$

$$y = e^{\alpha x}\left(c_1 \cos \beta x + c_2 \sin \beta x\right)$$ ②

$$y_c = e^{4x}\left(c_1 \cos 2x + c_2 \sin 2x\right)$$ ——— ⓑ

ui y_p ain $Q(x) = 100x^2 - 26xe^x$

Assume $y_p = (Ax^2 + Bx + C) + (Dx + E)e^x$

$$y' = 2Ax + B + (Dx + E)e^x + e^x D$$

$$y' = 2Ax + B + Dxe^x + Ee^x + De^x$$ ——— ①

$$y'' = 2A + D\left[xe^x + e^x\right] + Ee^x + De^x$$

$$y'' = 2A + Dxe^x + De^x + Ee^x + De^x$$

$$y'' = 2A + Dxe^x + 2De^x + Ee^x$$ ——— ②

sub ① ② — solution

$$2A + Dxe^x + 2De^x + Ee^x - 8\left(2Ax + B + Dxe^x + Ee^x + De^x\right) + 20\left[Ax^2 + Bx + \right.$$
$$\left. + (Dx + E)e^x\right] = 100x^2 - 26xe^x$$

$$2A + Dxe^{x} + \boxed{2De^{x}} + Ee^{x} - 16Ax - 8B - 8Dxe^{x} - 8Ee^{x} - 8De^{x} + 20Ax^{2} + 20Bx$$

$$+ 20C + 20Dxe^{x} + 20Ee^{x} = 100x^{2} - 26xe^{x}$$

$$13Dxe^{x} - 6De^{x} + 13Ee^{x} + (-16A + 20B)x + 20Ax^{2} + 2A - 8B + 20C$$

$$= 100x^{2} - 26xe^{x}$$

[illegible]

$$20A = 100$$

$$A = \frac{100}{20} = 5$$

$$13D = -26$$

$$D = \frac{-26}{13} = -2$$

$$-16A + 20B = 0$$

$$-16(5) + 20B = 0$$

$$-80 + 20B = 0$$

$$B = \frac{80}{20} = 4$$

$$2A - 8B + 20C = 0$$

$$2(5) - 8(4) + 20C = 0$$

$$C = \frac{-10+32}{20} = \frac{22}{20} = \frac{11}{10}$$

$$(-6D + 13E) = 0$$

$$-6(-2) + 13E = 0$$

$$E = -\frac{12}{13}$$

$$y_p = 5x^2 + 4x + \frac{11}{10} + \left(-2x - \frac{12}{13}\right)e^{x}$$

$$y = y_c + y_p$$

$$y = e^{4x}\left(c_1 \cos 2x + c_2 \sin 2x\right) + 5x^2 + 4x + \frac{11}{10} + \left(-2x - \frac{12}{13}\right)e^{x}$$

ข้อสอบจริง คำตอบ $y = c_1 e^{3x} + c_2 e^{-3x} - 6$ (1)

(7)

$$y'' - 9y = 54$$

สมการช่วย

$$(m^2 - 9)y = 0$$

$$m^2 - 9 = 0$$

$$m_1, m_2 = \sqrt{9} = \pm 3$$

$$y_c = c_1 e^{3x} + c_2 e^{-3x}$$

$D \rightarrow$ เป็น ตัวกำจัด 54

จะได้

$$(D^2 - 9)y = 54$$

$$D(D^2 - 9)y = D\,54 = 0$$

$$D(D^2 - 9)y = 0$$

สมการช่วย

$$m(m^2 - 9) = 0$$

$$m_1 = 0$$

$$m_2, m_3 = +3, -3$$

$$y = c_1 + \underbrace{c_2 e^{3x} + c_3 e^{-3x}}_{\text{ซ้ำ}}$$

$$\therefore \quad y_p = c_1 = A$$

$$y_p' = \frac{dA}{dx} = 0, \quad y_p'' = 0$$

แทนค่า y_p , y_p' , y_p'' ลงในโจทย์

$$0 - 9(A) = 54$$

$$A = \frac{54}{-9} = -6$$

$$\therefore \quad y_p = -6$$

ดัง $y = y_c + y_p$

$$y = c_1 e^{3x} + c_2 e^{-3x} - 6$$

(4)

ถ้าว $y = c_1\cos x + c_2\sin x + \cos x \ln|\cos x| + x\sin x$ (1)

— เป็นผลเฉลยของ

$$y'' + y = \sec x$$

สมการช่วย

$$(m^2+1)y = 0$$

$$(m^2+1) = 0$$

$$m^2 = -1$$

$$m = \sqrt{-1} = \pm j = 0 \pm j = a \pm bj$$

$$m_1 = i, \quad m_2 = -i$$

$$y_c = e^{ax}(c_1\cos bx + c_2\sin bx)$$

$$y_c = e^{0}(c_1\cos x + c_2\sin x)$$

$$y_c = c_1\cos x + c_2\sin x$$

หา y_p

$$y_p = u_1\cos x + u_2\sin x \quad (2)$$

$y_1(x) = \cos x$, $\quad y_2(x) = \sin x$

$y_1'(x) = -\sin x \qquad y_2'(x) = \cos x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$W = \cos^2 x + \sin^2 x = 1$$

จัดรูป เขียน form $y'' + py' + qy = f(x)$

$$f(x) = \sec x$$

$$v_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & \sin x \\ \sec x & \cos x \end{vmatrix}$$

$$v_1 = -\sin x \sec x$$

$$w_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} \cos x & 0 \\ -\sin x & \sec x \end{vmatrix}$$

$$w_2 = \cos x \sec x$$

$$U_1' = \frac{u_1}{w} = \frac{-\sin x \sec x}{+1} = -\sin x \sec x$$

$$U_1 = \int -\sin x \sec x \, dx = -\int \frac{\sin x}{\cos x} dx =$$

$$U_1 = -\int \tan x \, dx = \cancel{\ln \sec x} - (-\ln \operatorname{cosec} x)$$

$$U_1 = \ln \operatorname{cosec} x \quad \text{———} \quad (1)$$

$$U_2' = \frac{U_2}{w} = \underbrace{\left[\frac{\cos x}{\cos x}\right]}_{1} = 1$$

$$U_2 = \int 1 \, dx = x \quad \text{———} \quad (2)$$

using (1) (2) in (3)

$$y_p = \cos x \ln \operatorname{cosec} x + x \sin x$$

$$y = y_p + y_c = c_1 \cos x + c_2 \sin x + \cos x \ln|\operatorname{cosec} x| + x \sin x$$

[illegible]

[illegible]

[illegible]

$$(D^2+4D+3)y = 0$$

[illegible]

$$(m^2+4m+3) = 0$$

$$(m+1)(m+3) = 0$$

$$y_c = c_1 e^{-x} + c_2 e^{-3x}$$

[illegible]

$$\frac{1}{D-m} f(x) = e^{mx} \int e^{-mx} f(x)\, dx$$

[illegible]

$$(D^2+4D+3)y = 0$$

$$(D+3)(D+1)\,y = 0$$

$$y_p = \frac{1}{(D+3)(D+1)}(0)$$

$$y_p = 0$$

$$\therefore\ y = c_1 e^{-x} + c_2 e^{-3x}$$

6

$$(D-2)(D^2+2D+10)y = 0$$

สมการช่วย

$$(m-2)(m^2+2m+10) = 0$$

$$m_1 = 2$$

$$m_{2,3} = \frac{-2 \pm \sqrt{2^2-4(1)(10)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4-40}}{2} = \frac{-2 \pm \sqrt{-36}}{2}$$

$$= \frac{-2 \pm 6\sqrt{-1}}{2} = -1 \pm 3i = a+bi$$

$$y_c = c_1 e^{2x} + e^{-x}(c_2 \cos 3x + c_3 \sin 3x)$$

$$y_c = c_1 e^{2x} + c_2 e^{-x} \cos 3x + c_3 e^{-x} \sin 3x$$

$$y_p = 0$$