Abu Jouma Math 4 (1)

$$y = c_1 e^{-x} + c_2 e^{3x} + c_3 x e^{3x}$$

(1)

$$y''' - 5y'' + 3y' + 9y = 0$$

$$m^3 - 5m^2 + 3m + 9 = 0$$

$m = 1 \rightarrow 1 - 5 + 3 + 9 \neq 0$

$m = -1 \rightarrow -1 - 5 - 3 + 9 =$

$\cancel{1}$	1	-5	3	9
		1	-4	-1
	1	-4	-1	8

-1	1	-5	3	9
		-1	6	-9
	1	-6	9	0

m^3 m^2

$$(m+1)(m^2 - 6m + 9) = 0$$

$$(m+1)(m-3)(m-3) = 0$$

$$m = -1, 3, 3$$

$$y_c = c_1 e^{-x} + c_2 e^{3x} + c_3 x e^{3x}$$

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(2) $$y'' - 5y' = 2x^3 - 4x^2 - x + 6 \quad \text{———} \quad (1)$$

$$y = C_1 e^{5x} + C_2 - \frac{1}{10}x^4 + \frac{14}{75}x^3 + \frac{53}{250}x^2 - \frac{647}{625}x$$

หา y_c

$$y'' - 5y' = 0$$

$$m^2 - 5m = 0$$

$$m(m-5) = 0$$

$$m = 0, +5$$

$$y_c = C_1 e^{0x} + C_2 e^{5x} = C_1 + C_2 e^{5x}$$

จาก $g(x) = 2x - x^2 \cdots$

ดังนั้น $$y_p = Ax^3 + Bx^2 + Cx + D \quad \text{———} \quad (2)$$

$$y' = 3Ax^2 + 2Bx + C \quad \text{———} \quad (3)$$

$$y'' = 6Ax + 2B \quad \text{———} \quad (4)$$

แทน (3) (4) ลงใน (1)

$$(6Ax + 2B) - 5(3Ax^2 + 2Bx + C) = 2x^3 - 4x^2 - x + 6$$

สมมุติ y_p ใหม่ คูณด้วย x^n → n พอดีที่จะไม่ได้ไม่เหมือนเดิมที่

$$\cancel{y_p = Ax^4 + Bx^3 + Cx^2 + Dx + E}$$

$$y_p = Ax^4 + Bx^3 + Cx^2 + Dx$$

$$y' = 4Ax^3 + 3Bx^2 + 2Cx + D \quad \text{———} \quad (2)$$

$$y'' = 12Ax^2 + 6Bx + 2C \quad \text{———} \quad (3)$$

แทน (2) (3) ใน (1)

$$12Ax^2 + 6Bx + 2C - 5(4Ax^3 + 3Bx^2 + 2Cx + D) = 2x^3 - 4x^2 - x + 6$$

$$\underline{12Ax^2} + \underline{6Bx} + 2C \underline{- 20Ax^3} \underline{- 15Bx^2} \underline{-10Cx} - 5D = 2x^3 - 4x^2 - x + 6$$

$$-20Ax^3 + (12A - 15B)x^2 + (6B - 10C)x + (2C - 5D) = 2x^3 - 4x^2 - x + 6$$

จะได้ $\quad -20A = 2$

$$A = \frac{2}{-20} = -\frac{1}{10} \quad \text{———} \quad (4)$$

$$12A - 15B = -4$$

$$-15B = -4 - 12A$$

$$B = \frac{-4 - 12A}{-15} \longrightarrow \text{แทน } A.$$

$$B = \frac{-4 - 12\left(-\frac{1}{10}\right)}{-15} = \frac{-4 + 1.2}{-15} = 0.1866$$

$$= \frac{14}{75}$$

$(6B - 10C) = -1$

$6B - 10C = -1$

$6\left(\frac{14}{75}\right) - 10C = -1$

$-10C = -1 - 1.12$

$C = 0.212$

$C = \frac{53}{250}$ —— $\textcircled{b}$

$2C - 5D = 6$

$\frac{2 \times 53}{250} - 5D = 6$

$D = \frac{6 - 0.424}{-5}$

$D = -1.1152 = -\frac{697}{625}$

$y = y_c + y_p$

$= c_1 + c_2 e^{5x} - \frac{1}{10}x^4 + \frac{14}{75}x^3 + \frac{53}{250}x^2 - \frac{697}{625}x$

3

$$y'''' - 8y'' + 16y = (x^3 + 2x)e^x$$

สมการช่วย

$$m^4 - 8m^2 + 16 = 0$$

$$(m^2 - 4)(m^2 - 4)$$

$$m^2 - 4 = 0$$

$$m^2 = 4$$

$$m = \sqrt{4} = +2, -2, +2, -2$$

$$y_c = (c_1 + c_2 x)e^{2x} + (c_3 + c_4 x)e^{-2x}$$

จาก $g(x) = (x^3 + 2x)e^x$

จาก $(D-\alpha)^n \rightarrow$ เป็นตัวกำจัดของ $e^{\alpha x}, xe^{\alpha x}, \dots x^{n-1}e^{\alpha x}$

$$c_1 e^{\alpha x} + c_2 x e^{\alpha x} + \dots c_{n-1} x^{n-1} e^{\alpha x}$$

จาก $g(x)$ $\alpha = 1 \rightarrow n - 1 = 3$

$$n = 3 + 1 = 4$$

$(D-1)^4$ เป็นตัวกำจัด

จากโจทย์

$$(D-1)^4 (D^4 - 8D^2 + 16)y = \cancel{(D-1)^4 (x^3 + 2x)e^x} \; 0$$

$$(m-1)^4 (m^4 - 8m^2 + 16) = 0$$

$$m = 1, 1, 1, 1, +2, -2, +2, -2$$

$$y_c = (c_1 + c_2 x)e^{2x} + (c_3 + c_4 x)e^{-2x} + (c_5 + c_6 x + c_7 x^2 + c_8 x^3)$$

(4) $$y'' - 2y' + 2y = e^{x} \sec x$$

$$y = c_1 e^{x} \sin x + c_2 e^{x} \cos x + x e^{x} \sin x + e^{x} \cos x \ln|\cos x|$$

หาค่า. $$m^2 - 2m + 2 = 0$$

$$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$m = \frac{+2 \pm \sqrt{4-8}}{2} = \frac{+2 \pm 2j}{2}$$

$$m = 1 \pm 1j$$ → [illegible]

$$m_1 = \alpha + \beta j \qquad m_2 = \alpha - j\beta$$

$$y = c_1 e^{\alpha x} \cos \beta x + c_2 e^{\alpha x} \sin \beta x$$

$$y_c = c_1 e^{x} \cos x + c_2 e^{x} \sin x$$

[illegible]

$$y_c = c_1 e^{x} \sin x + c_2 e^{x} \cos x$$

[illegible]

$$y_p = u_1 e^{x} \sin x + u_2 e^{x} \cos x$$

$$y_p = u_1 y_1(x) + u_2 y_2(x)$$

$$y_1(x) = e^{x} \sin x$$

$$y_1'(x) = e^{x} \frac{d \sin x}{dx} + \sin x \frac{d e^{x}}{dx}$$

$$y_1'(x) = e^x \cos x + e^x \sin x$$

$$y_2(x) = e^x \cos x$$

sinx ⇒
csc

$$y_2'(x) = e^x \frac{d\cos x}{dx} + \cos x \frac{de^x}{dx}$$

$$y_2'(x) = -e^x \sin x + e^x \cos x$$

$$f(x) = e^x \sec x$$

$$w_1 = \begin{vmatrix} 0 & y_2 \\ f(x) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & e^x \cos x \\ e^x \sec x & -e^x \sin x + e^x \cos x \end{vmatrix}$$

$$w_1 = -e^{2x} \sec x \cos x = -e^{2x} \frac{\cancel{\cos x}^{\,1}}{\cancel{\cos x}} = -e^{2x}$$

$$w_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(x) \end{vmatrix} = \begin{vmatrix} e^x \sin x & 0 \\ e^x \cos x + e^x \sin x & e^x \sec x \end{vmatrix}$$

$$w_2 = e^{2x} \sin x \sec x = e^{2x} \frac{\sin x}{\cos x} = e^{2x} \tan x$$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x \sin x & e^x \cos x \\ e^x \cos x + e^x \sin x & -e^x \sin x + e^x \cos x \end{vmatrix}$$

$$w = -e^{2x} \sin^2 x + e^{2x} \sin x \cos x - e^{2x} \cos^2 x - e^{2x} \cancel{\sin x \cos x}$$

$$w = -e^{2x} (\underbrace{\sin^2 x + \cos^2 x}_{1}) = -e^{2x}$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$u_1' = \frac{W_1}{W} = \frac{-e^{2x}}{-e^{2x}} = 1$$

$$u_1 = \int 1\,dx = x$$

$$u_2' = \frac{W_2}{W} = \frac{e^{2x}\tan x}{-e^{2x}} = -\tan x$$

$$u_2 = \int -\tan x\,dx = -\ln \sec x$$

$$y_p = x e^{x} \sin x + -\ln \sec x \, e^{x} \cos x$$

$$y_p = x e^{x} \sin x - e^{x} \cos x \ln \sec x$$

$$y = y_c + y_p = c_1 e^{x} \sin x + c_2 e^{x} \cos x + x e^{x} \sin x - e^{x} \cos x \ln \sec x$$

$$y = c_1 e^{-3x} + c_2 e^{x} + 2e^{4x}$$

ตัวดำเนินการผกผัน

(5)

$$(D^2 + 2D - 3)y = 42e^{4x}$$

$$(D+3)(D-1)y = 42e^{4x}$$

$$y = \frac{1}{(D+3)(D-1)} 42e^{4x}$$

$$y = \frac{1}{(D+3)}\left[\frac{1}{(D-1)} 42e^{4x}\right]$$

$$(m+3)(m-1) = 0$$

$$y_c = c_1 e^{-3x} + c_2 e^{x}$$

$$m_1 = -3$$

$$m_2 = +1$$

จาก

$$\frac{1}{D-m} f(x) = e^{mx}\int e^{-mx} f(x)\,dx$$

$$\frac{1}{(D-1)} 42e^{4x} = 42\left[\frac{1}{(D-1)} e^{4x}\right]$$

$$= 42\left[e^{x}\int e^{-x} e^{4x}\,dx\right]$$

$$= 42\left[e^{x}\int e^{3x}\,dx\right]$$

$$= 42\left[\frac{1}{3} e^{x} e^{3x}\right]$$

$$= 14e^{4x}$$

$$y = y_c + y_p$$

$$y = c_1 e^{-3x} + c_2 e^{x} + 2e^{4x}$$

$$\frac{1}{(D+3)}\left(14e^{4x}\right) = e^{-3x}\int e^{3x}\, 14e^{4x}\,dx = 14e^{-3x}\int e^{7x}\,dx$$

$$y_p = \frac{14}{7} e^{-3x} e^{7x} = 2e^{4x}$$

$\mathcal{L}\, y''' = s^3 Y(s) - s^2 y(0) - s\, y'(0) - y''(0)$

$\mathcal{L}\, y'' = s^2 Y(s) - s\, y(0) - y'(0)$

$\mathcal{L}\, y' = s Y(s) - y(0)$

ii

$$\frac{d^2y}{dt^2} + 7\frac{dy}{dt} - 10y = 4t\, e^{-3t}$$

$$y(0) = 0 \quad , \quad y'(0) = -1$$

$$y'' + 7y' - 10y = 4t e^{-3t}$$

$$\mathcal{L} y'' + 7\mathcal{L} y' - 10\mathcal{L} y = \mathcal{L}\, 4t e^{-3t}$$

$$\left[s^2 Y(s) - s \overset{0}{y(0)} - \overset{-1}{y'(0)}\right] + 7\left[s Y(s) - \overset{0}{y(0)}\right]$$

$$-10\,Y(s) = 4\mathcal{L}\, t e^{-3t} = 4\,\frac{1}{(s+3)^2}$$

$$= \frac{4}{(s+3)^2}$$

$$s^2 Y(s) + 1 + 7s Y(s) - 10\, Y(s) = \frac{4}{(s+3)^2}$$

$$(s^2 + 7s - 10)\, Y(s) + 1$$

$$(s^2 + 7s - 10)\, Y(s) = \frac{4}{(s+3)^2} - 1 = \frac{4 - (s+3)^2}{(s+3)^2}$$

$$Y(s) = \cancel{4 - s^2 - 6s + 9} \quad \frac{4 - (s^2 + 6s + 9)}{(s+3)^2 (s^2 + 7s - 10)}$$

$$Y(s) = \frac{4 - s^2 - 6s - 9}{(s+3)^2 (s^2 + 7s - 10)}$$

$$Y(s) = \frac{-s^2 - 6s - 5}{(s+3)^2 (s^2 + 7s - 10)} = \frac{A}{(s+3)^2} + \frac{B}{(s+3)} + \frac{Cs + D}{(s^2 + 7s - 10)}$$

$$-s^2 - 6s - 5 = A(s^2 + 7s - 10) + B(s+3)(s^2 + 7s - 10) + (Cs + D)(s+3)^2$$

$$-s^2 - 6s - 5 = A(s^2 + 7s - 10) + B(s^3 + 7s^2 - 10s + 3s^2 + 21s - 30) + (Cs + D)(s^2 + 6s + 9)$$

$$\cancel{= As^2 + 7As - 10A + Bs^3 + 7Bs^2 - 10Bs + 3Bs^2 + 12}$$

$$= A(s^2 + 7s - 10) + B(s^3 + 10s^2 + 11s - 30) + [Cs^3 + 6Cs^2 + 9Cs +$$

$$+ 6Ds + 9D]$$

$$= As^2 + 7As - 10A + Bs^3 + 10Bs^2 + 11Bs - 30B + Cs^3 + 6Cs^2 + 9$$

$$+ Ds^2 + 6Ds + 9D$$

$$= (B + C)s^3 + (A + 10B + 6C + D)s^2 + (7A + 11B + 9C + 6D$$

$$+(-10A - 30B + 9D)$$

$B + C = 0$ ——— (1)

$A + 10B + 6C + D = -1$ ——— (2)

$7A + 11B + 9C + 6D = -6$ ——— (3)

$-10A - 30B + 9D = -5$ ——— (4)

[illegible] (1) $\cancel{B = -C}$ $C = -B$ ——— (5)

[illegible] (5) [illegible] (2)

$$A + 10B + 6(-B) + D = -1$$

$$A + 10B - 6B + D = -1$$

$A + 4B + D = -1$ ——— (6)

[illegible] (5) [illegible] (3)

$$7A + 11B + 9(-B) + 6D = -6$$

$$7A + 11B - 9B + 6D = -6$$

$7A + 2B + 6D = -6$ ——— (7)

[illegible] (4), (6), (7)

$$-10A - 30B + 9D = -5$$

$$A + 4B + D = -1$$

$$7A + 2B + 6D = -6$$

$$\text{Det} = \begin{bmatrix} -10 & -30 & 5 \\ 1 & 4 & 1 \\ 7 & 2 & 6 \end{bmatrix} * \begin{bmatrix} A \\ B \\ D \end{bmatrix} = \begin{bmatrix} -5 \\ -1 \\ -6 \end{bmatrix}$$

$$\text{Det} = \begin{bmatrix} -10 & -30 & 5 \\ 1 & 4 & 1 \\ 7 & 2 & 6 \end{bmatrix} \begin{matrix} -10 & -30 \\ 1 & 4 \\ 7 & 2 \end{matrix}$$

$$= \left[-240 - 210 + 18\right] + \left[-252 + 20 + 180\right] = -484$$

$$A = \frac{\begin{bmatrix} -5 & -30 & 5 \\ -1 & 4 & 1 \\ -6 & 2 & 6 \end{bmatrix} \begin{matrix} -5 & -30 \\ -1 & 4 \\ -6 & 2 \end{matrix}}{-484} = \frac{-120 + 180 - 18 + 216 + 10 - 180}{-484}$$

$$A = \frac{88}{484} = \frac{22}{121} = \frac{22}{121}$$

$$B = \begin{bmatrix} -10 & -5 & 5 \\ 1 & -1 & 1 \\ 7 & -6 & 6 \end{bmatrix} \begin{matrix} -10 & -5 \\ 1 & -1 \\ 7 & -6 \end{matrix}$$

$$= \frac{+60 - 35 - 54 + 63 - 60 + 30}{-484}$$

$$B = \frac{4}{-484} = -\frac{1}{121}$$

$$D = \begin{bmatrix} -10 & -30 & -5 \\ 1 & 4 & -1 \\ 7 & 2 & -6 \end{bmatrix} \begin{matrix} -30 & -5 \\ 4 & -1 \\ 2 & -6 \end{matrix}$$

$$= +240 + 60 + 120 + 140 - 60 - 120$$

$$B = \frac{380}{-484} = \frac{95}{-121} = -\frac{95}{121}$$

$$C = -B = +\frac{1}{121}$$

$$Y(s) = \frac{22}{121}\,\frac{1}{(s+3)^2} - \frac{1}{121}\,\frac{1}{(s+3)} + \frac{\frac{1}{121}s - \frac{95}{121}}{(s^2+7s-10)}$$

a) $y''' - 5y'' + 7y' - 3y = 20\sin t$

$y(0) = 0 \quad y'(0) = 0 \quad y''(0) = -2$

$\mathcal{L}[\quad \longrightarrow \quad]$

$$s^3Y(s) - s^2\cancel{y(0)}^{0} - s\cancel{y'(0)}^{0} - \cancel{y''(0)}^{-2} - 5\left[s^2Y(s) - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0}\right]$$

$$+7\left[sY(s) - \cancel{y(0)}^{0}\right] - 3Y(s) = 20\left[\frac{1}{s^2+1}\right) = \frac{20}{s^2+1}$$

$$s^3Y(s) + 2 - 5s^2Y(s) + 7sY(s) - 3Y(s) = \frac{20}{s^2+1}$$

$$s^3Y(s) - 5s^2Y(s) + 4Y(s) + 2 = \frac{20}{s^2+1}$$

$$Y(s)(s^3 - 5s^2 + 4) + 2 = \frac{20}{s^2+1}$$

$$Y(s)(s^3 - 5s^2 + 4) = \frac{20}{s^2+1} - 2 = \frac{20 - 2(s^2+1)}{s^2+1}$$

$$= \frac{20 - 2s^2 - 2}{s^2+1} = \frac{-2s^2+18}{s^2+1}$$

$$Y(s) = \frac{-2s^2+18}{(s^2+1)(s^3-5s^2+4)} = \cancel{\frac{-2s^2+18}{(s^2+1)(s^2-4)(s-1)}}$$

$$\cancel{Y(s) = \frac{A}{(s-4)} + \frac{B}{(s-1)} + \frac{Cs+D}{(s^2+1)}}$$

$$\cancel{-2s^2+18 = A(s-1)(s^2+1) + B(s-?)(s^2+1) + (Cs+D)(s^2}$$

$$\cancel{(s^2 \quad)(s}$$

$$s^3 Y(s) + 2 - 5s^2 Y(s) + 7s Y(s) - 3Y(s) = \frac{20}{s^2+1}$$

$$Y(s)\left[s^3 - 5s^2 + 7s - 3\right] + 2 = \frac{20}{s^2+1}$$

$$Y(s)\left[s^3 - 5s^2 + 7s - 3\right] = \frac{20}{s^2+1} - 2 = \frac{20 - 2(s^2+1)}{s^2+1}$$

$$Y(s)\left[s^3 - 5s^2 + 7s - 3\right] = \frac{20 - 2s^2 - 2}{s^2+1} = \frac{-2s^2+18}{s^2+1}$$

$$Y(s) = \frac{-2s^2+18}{(s^2+1)(s^3-5s^2+7s-3)}$$

$$Y(s) = \frac{-2s^2+18}{s^5 - 5s^4 + 7s^3 - 3s^2 + s^3 - 5s^2 + 7s - 3}$$

$$Y(s) = \frac{-2s^2+18}{s^5 - 5s^4 + 8s^3 - 8s^2 + 7s - 3}$$

$-1 \quad -0$

$$\frac{-1 \quad -5 \quad -8 \quad -8 \quad \cancel{8} -7 \quad -3}{1 \quad -5 \quad \cancel{+8} \quad \cancel{-8} \quad \cancel{+7} \quad \cancel{-3}}$$

$v \qquad 4$

s^5 1	1	−5	+8	−8	+7	−3
		1	−4	4	−4	+3
s^4 3	1	−4	4	−4	3	0 ✓
		3	−3	3	−3	
	1	−1	1	−1	0 ✓	

$(s-1)(s-3)(s^3 - s^2 + s - 1)$

$f(s)$:

$s^2(s-1) + (s-1)$

$(s-1)^2 = s^2 - 2s + 1$

$(s^2 - 2s + 1)(s+1)$ =

$\cancel{s^3 - 2s^2 + s - s^2 + 2s - 1}$

$\cancel{s^3 - 3s^2 + 3s - 1}$

$s^3 - 2s^2 + s + s^2 - 2s + 1$

$s^3 - s^2 - s + 1$

$(s+1)^2(s-1) = (s^2 + 2s + 1)(s-1)$

$= s^3 + 2s^2 + s - s^2 - 2s - 1$

$= s^3 + s^2 - s - 1$