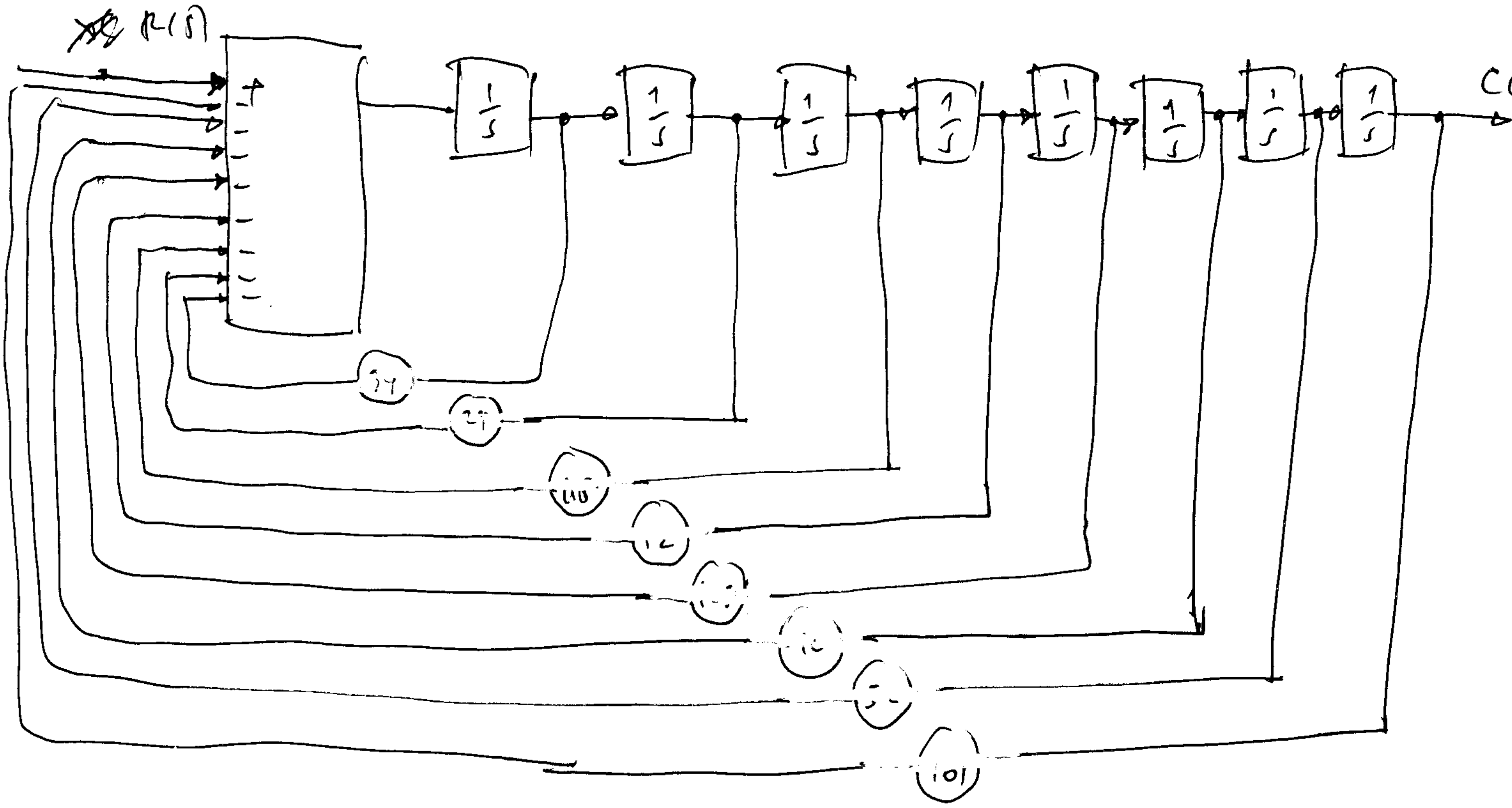
เฉลย ปัญหาพิเศษ 2/2559 ①

feed back

① จาก $T(s) = \dfrac{1}{s^8 + 74s^7 + 24s^6 + 115s^5 + 12s^4 + 108s^3 + 10s^2 + 32s + \ldots}$

จงหา รูปแบบ [illegible]

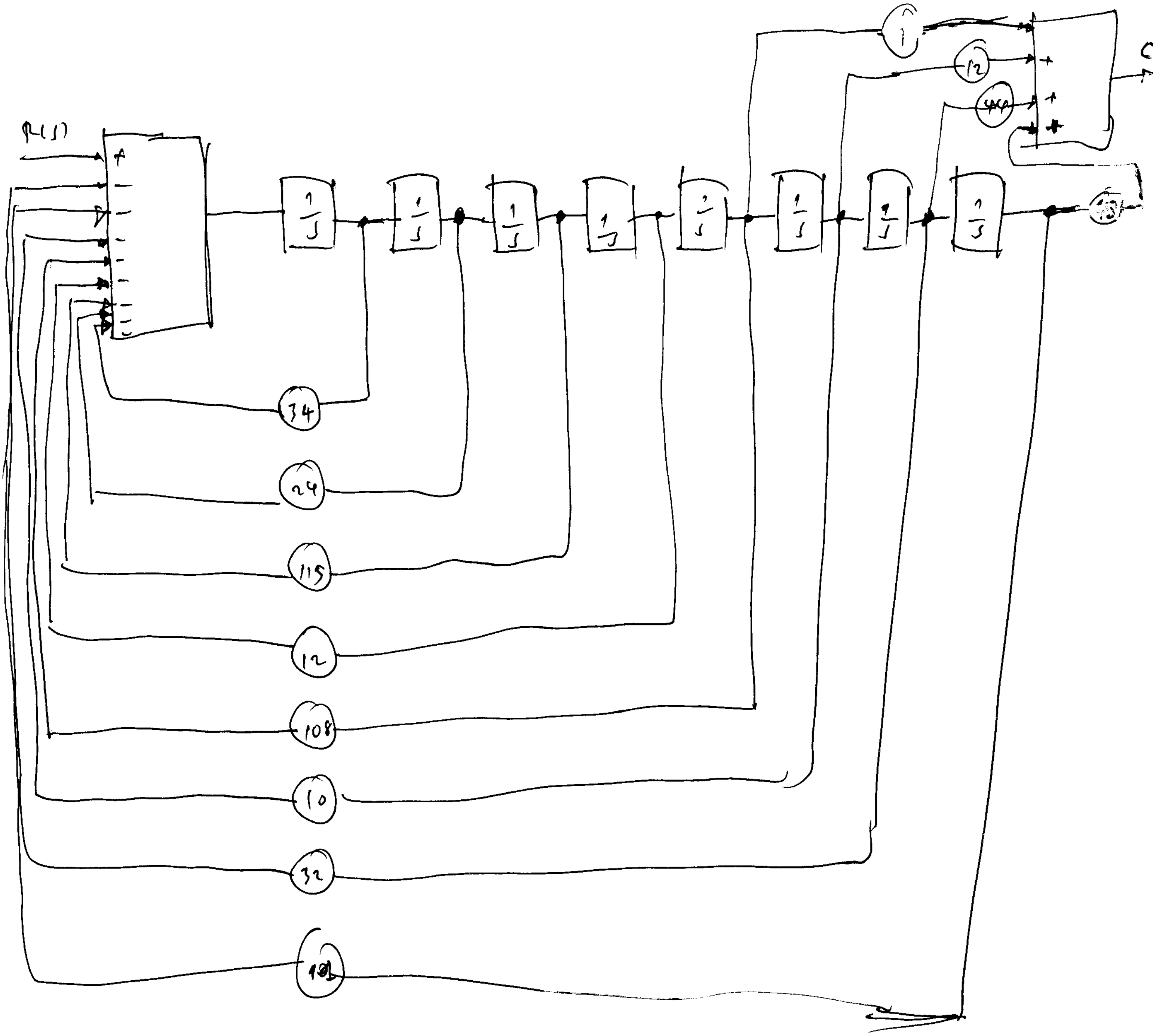
$$T(s) = \frac{1}{(((((((s+74)s + 24)s + 115)s + 12)s + 108)s + 10)s + 32)s + \ldots}$$



(1)

(2)

$$\frac{C(s)}{R(s)} = T(s) = \frac{s^3+12s^2+44s+48}{s^8+34s^7+24s^6+115s^5+12s^4+108s^3+10s^2+32s+101}$$



③ ใช้ ลีนาร์ด-ชิปาร์ต

$$P(s) = s^5 + s^4 + 6s^3 + 5s^2 + 12s + 20$$

$n = 5 \rightarrow$ คี่ ใช้เงื่อนไข

$$a_n > 0\,,\ a_{n-2} > 0\,, \ldots\ \Delta_2 > 0\,,\ \Delta_4 > 0\,, \ldots$$

หรือใช้ ~~ความสัมพันธ์~~ ความสัมพันธ์

$$a_n > 0\,,\ a_{n-2} > 0\,, \ldots\ \Delta_2 > 0\,,\ \Delta_4 > 0\,, \ldots$$

$$\Delta_i = \begin{vmatrix} a_1 & a_3 & a_5 & \cdots \\ 1 & a_2 & a_4 & \cdots \\ 0 & a_1 & a_3 & \cdots \\ 0 & 1 & a_2 & \cdots \\ & \cdots & & a_i \end{vmatrix}$$

$$P(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n$$

$a_0 = 1$, $a_1 = 1$, $a_2 = 6$, $a_3 = 5$, $a_4 = 12$, $a_5 = 20$

$$\Delta_i = \begin{bmatrix} 1 & 5 & 20 & 0 \\ 1 & 6 & 12 & 0 \\ 0 & 1 & 5 & 20 \\ 0 & 1 & 6 & 12 \end{bmatrix}$$

②

n = 5 → เป็นเลขคี่

$a_1 = 1 > 0$

$a_2 = 6 > 0$

$a_3 = 5 > 0$

$$\Delta_2 = \begin{bmatrix} 1 & 5 \\ 1 & 6 \end{bmatrix} = 6 - 5 = 1 > 0$$

$$\Delta_4 = \left|\begin{matrix} 1 & 5 & 20 & 0 \\ 1 & 6 & 12 & 0 \\ 0 & 1 & 5 & 20 \\ 0 & 1 & 6 & 12 \end{matrix}\right| \begin{matrix} 1 & 5 & 20 \\ 1 & 6 & 12 \\ 0 & 1 & 5 \\ 0 & 1 & 6 \end{matrix} =$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \begin{matrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \end{matrix}$$

an Δ. von Matrix

$$\left|\begin{matrix} 2 & 4 & 10 & 0 \\ 1 & 2 & 11 & 0 \\ 0 & 2 & 4 & 10 \\ 0 & 1 & 2 & 11 \end{matrix}\right| \begin{matrix} 2 & 4 & 10 \\ 1 & 2 & 11 \\ 0 & 2 & 4 \\ 0 & 1 & 2 \end{matrix} = 196 - 60 =$$

$$\Delta_4 = \begin{pmatrix} 1 & 5 & 20 & 0 \\ 1 & 6 & 12 & 0 \\ 0 & 1 & 5 & 20 \\ 0 & 1 & 6 & 12 \end{pmatrix}$$

$$1(-1)\left[\begin{array}{ccc|cc} 1 & 20 & 0 & 1 & 20 \\ 1 & 12 & 0 & 1 & 12 \\ 0 & 6 & 12 & 0 & 6 \end{array}\right] = (-1)(144-240)$$

$$+\ 5(1)\left[\begin{array}{ccc|cc} 1 & 5 & 0 & 1 & 5 \\ 1 & 6 & 0 & 1 & 6 \\ 0 & 1 & 12 & 0 & 1 \end{array}\right] = (5)(72-60)$$

$$+\ 20(-1)\left[\begin{array}{ccc|cc} 1 & 5 & 20 & 1 & 5 \\ 1 & 6 & 12 & 1 & 6 \\ 0 & 1 & 6 & 0 & 1 \end{array}\right] = (-20)(36+20-12-30)$$

$$\Delta_4 = 96 + 60 - 280 = -124 < 0$$

[illegible]

4

[illegible] [illegible]

$$\Delta_4 = \begin{vmatrix} 2 & 4 & 10 & 0 \\ 1 & 2 & 11 & 0 \\ 0 & 2 & 4 & 10 \\ 0 & 1 & 2 & 11 \end{vmatrix} = -144$$

$$-2 \begin{bmatrix} 2 & 10 & 0 \\ 1 & 11 & 0 \\ 0 & 2 & 11 \end{bmatrix} \begin{matrix} 2 & 10 \\ 1 & 11 \\ 0 & 2 \end{matrix} = (-2)(242 - 110)$$

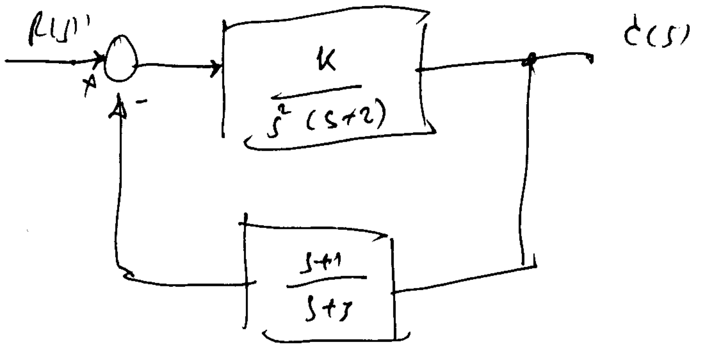
$$4 \begin{bmatrix} 2 & 4 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 11 \end{bmatrix} \begin{matrix} 2 & 4 \\ 1 & 2 \\ 0 & 1 \end{matrix} = (4)\overset{0}{\cancel{(44 - 44)}}$$

$$-10 \begin{bmatrix} 2 & 4 & 10 \\ 1 & 2 & 11 \\ 0 & 1 & 2 \end{bmatrix} \begin{matrix} 2 & 4 \\ 1 & 2 \\ 0 & 1 \end{matrix} = (-10)(8 + 10 - 22 - 8)$$

$$= -264 + 120 = -144$$

4

דרך ראשונה



$$\frac{C(s)}{R(s)} = \frac{\left[\frac{K}{s^2(s+2)}\right]}{1+\left[\frac{s+1}{s+3}\right]\left[\frac{K}{s^2(s+2)}\right]}$$

$$\cancel{\frac{C(s)}{R(s)} = \frac{}{[s+3][s^2(s+2)] + (s+1)(s+4)K}}$$

$$\frac{C(s)}{R(s)} = \frac{\left[\frac{K}{s^2(s+2)}\right]}{\left[\frac{(s+3)s^2(s+2) + (s+1)K}{(s+3)s^2(s+2)}\right]}$$

$$= \frac{K}{\cancel{s^2(s+2)}} * \frac{(s+3)\cancel{s^2(s+2)}}{(s+3)s^2(s+2) + (s+1)K}$$

$$= \frac{Ks+3K}{(s^3+3s^2)(s+2) + Ks + K}$$

$$= \frac{ks + 3k}{s^4 + 2s^3 + 3s^3 + 6s^2 + ks + k}$$

$$\frac{C(s)}{R(s)} = \frac{ks + 3k}{s^4 + 5s^3 + 6s^2 + ks + k}$$

$n = 4 \rightarrow$

$$P(s) = s^4 + 5s^3 + 6s^2 + ks + k$$

$a_1 = 5$, $a_2 = 6$, $a_3 = k$, $a_4 = k$

$a_n > 0$, $a_{n-2} > 0$, ... ; $\Delta_1 > 0$, $\Delta_3 > 0$

$$\Delta_i = \begin{vmatrix} 5 & k & 0 & 0 \\ 1 & 6 & k & 0 \\ 0 & 5 & k & 0 \\ 0 & 1 & 6 & k \end{vmatrix}$$

Δ_1, Δ_2, Δ_3

$$\Delta_1 = 5 > 0$$

$$\Delta_3 = \begin{vmatrix} 5 & k & 0 \\ 1 & 6 & k \\ 0 & 5 & k \end{vmatrix} \begin{matrix} 5 & k \\ 1 & 6 \\ 0 & 5 \end{matrix}$$

$+ \quad + \quad +$

$$\Delta_3 = 30k - 25k - k^2 > 0$$

$$5k - k^2 > 0$$

$$-k^2 > -5k$$

$$-k > -5$$

$$\cancel{k > 5} \quad k < 5$$

$$5k > k^2$$

$$5 > \frac{k^2}{k}$$

$$5 > k$$

$$k < 5$$

จาก $30k - 25k - k^2 > 0$

ถ้า $k = 4 \rightarrow 30(4) - 25(4) - 4^2 > 0$

$$120 - 100 - 16 > 0$$

$$120 - 116 > 0$$

$$4 > 0 \checkmark$$

ถ้า $k = 6 \rightarrow 30(6) - 25(6) - 6^2 > 0$

สรุป $k < 5$ $\qquad 180 - 150 - 36 > 0$

ระบบถึงจะเสถียร $\qquad -6 > 0$ — ไม่เป็นจริง

ข้อ 5 จาก function ถ่ายโอนของระบบ

$$T(s) = \frac{20}{s^2 + 3s + 20}$$

บอกค่า ซีต้า เท่าใด เทียบกับเท่าใด

สมการคุณลักษณะ $s^2 + 3s + 20$

เทียบกับสมการของระบบอันดับสองมาตรฐาน

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

จาก

$$\omega_n^2 = 20$$

$$\omega_n = \sqrt{20}$$

$$\omega_n = 4.472$$

$$2\zeta\omega_n = 3$$

$$2\sqrt{20}\,\zeta = 3$$

$$\zeta = \frac{3}{2\sqrt{20}} = 0.335$$

$$\theta = \cos^{-1}\zeta = \cos^{-1} 0.335$$

~~θ = 1.228~~ $\theta = 70.40^\circ$

~~θ = 1.229~~

~~求~~ rise time

$$t_r = \frac{\pi - \theta}{\omega_n\sqrt{1-\zeta^2}}$$

$$t_r = \frac{\pi - \cancel{1.26} \frac{78.40 \times \pi}{180}}{\omega_d}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 4.472\sqrt{(1-\cancel{0.35}\,0.335^2)}$$

$$\omega_d = 4.213$$

$$t_r = \frac{1.77}{4.213} = 0.420 \quad \text{second}$$ —— #

~~$t_r = 42\ \%$~~

$$t_p = \frac{\pi}{\omega_d} = \cancel{\frac{3.14}{4.22}} = \frac{3.141}{4.213} = 0.7456$$ —— #

$$t_p = 0.7456 \quad \text{second.}$$ —— #

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \times 100\% = e^{-\frac{\pi \times 0.335}{\sqrt{1-0.335^2}}}$$

$$M_p = e^{-1.1884} = 0.3056$$

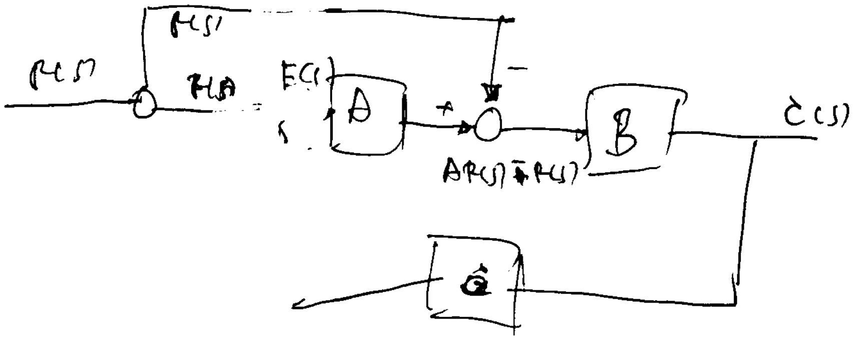
$$M_p = 30.56\ \%$$ —— #

$$t_s(5\%) \cong \frac{3}{\zeta \omega_n}$$

$$t_s(5\%) = \frac{3}{0.335 \times 4.472}$$

$$t_s(5\%) \cong \frac{3}{1.498} = 2.002 \text{ sec}$$

6 ลดทอนบล็อกของระบบ.



~~forward path~~

$$\cancel{[AR(s) - R(s)]B = C(s)}$$

$$\cancel{ABR(s) - BR(s) = C(s)}$$

$$E(s) = R(s) - G\,C(s) \quad \text{———} \quad (1)$$

$$[AE(s) - R(s)]B = C(s)$$

$$ABE(s) - BR(s) = C(s) \quad \text{———} \quad (2)$$

แทน (1) ใน (2)

$$AB[R(s) - G\,C(s)] - BR(s) = C(s)$$

$$ABR(s) - ABG\,C(s) - BR(s) = C(s)$$

$$(AB - B)R(s) = (1 + ABG)\,C(s)$$

$$\frac{C(s)}{R(s)} = \frac{(AB - B)}{(1 + ABG)} \quad \text{———} \quad (3)$$

Where $A = \frac{4}{(s+3)}$, $B = \frac{3}{s+4}$, $G = \frac{2}{s+5}$

$$\frac{C(s)}{R(s)} = \frac{\left[\frac{4}{(s+3)} \times \frac{3}{(s+4)} - \frac{3}{(s+4)}\right]}{\left[1 + \frac{4}{(s+3)} \frac{3}{(s+4)} \frac{2}{(s+5)}\right]}$$

$$\frac{C(s)}{R(s)} = \frac{\left[\frac{12 - 3(s+3)}{\cancel{(s+3)}\cancel{(s+4)}}\right]}{\left[\frac{(s+3)(s+4)(s+5) + 24}{\cancel{(s+3)}\cancel{(s+4)}(s+5)}\right]}$$

$$\frac{C(s)}{R(s)} = \frac{(s+5)\left[12 - 3(s+3)\right]}{(s+3)(s+4)(s+5) + 24}$$

$$\frac{C(s)}{R(s)} = \frac{12(s+5) - 3(s+5)(s+3)}{(s+3)(s+4)(s+5) + 24}$$

$$= \frac{12s + 60 - 3(s^2 + 3s + 5s + 15)}{(s^2 + 4s + 3s + 12)(s+5) + 24}$$

$$\frac{Z(s)}{R(s)} = \frac{12s + 60 - 3s^2 - 9s - 15s - 30}{s^3 + 4s^2 + 3s^2 + 12s + 5s^2 + 20s + 15s + 60 + 24}$$

$$\frac{Z(s)}{R(s)} = \frac{-3s^2 - 12s + 30}{s^3 + 12s^2 + 47s + 84}$$

$$\cancel{\frac{Z(s)}{R(s)}} =$$

$$T(s) = \left(\underline{\qquad\qquad} \right)$$

$$\cancel{E(s) = T}$$

$$T_E(s) = 1 - T(s)$$

$$= 1 - T(s) = 1 - \frac{-3s^2 - 12s + 30}{s^3 + 12s^2 + 47s + 84}$$

$$T_E(s) = \frac{s^3 + 12s^2 + 47s + 84 + 3s^2 + 12s - 30}{s^3 + 12s^2 + 47s + 84}$$

$$T_E(s) = \frac{s^3 + 15s^2 + 59s + 54}{s^3 + 12s^2 + 47s + 84}$$

$$E(s) = T_E(s)\,R(s)$$

ใช้ทฤษฎีค่าสุดท้าย

ดังนั้น

$C_0 = \frac{23}{28}, \quad C_1 = \infty, \quad C_2 = \infty$

$$e_{ss} = \lim_{s \to 0} s\, T_E(s)\, R(s)$$

1. เมื่อสัญญาณอินพุตเป็นขั้นบันได

$$e_{ss} = \lim_{s \to 0} \cancel{s}\, \frac{1}{\cancel{s}}\, T_E(s) = T_E(0)$$

$$e_{ss} = \frac{0^2 + 15(0)^2 + 54(0) + 54}{0^3 + 12(0)^2 + 47(0) + 84}$$

$$e_{ss} = \frac{54}{84} = \frac{27}{42}$$

2. เมื่อสัญญาณอินพุตแบบลาดเอียง

$$C_1 = \lim_{s \to 0} \frac{T_E(s)}{s}$$

$$= \lim_{s \to 0} \frac{1}{\cancel{s}}\, \frac{\cancel{s}\left[(s^2 + 15s + 54) + \frac{54}{s}\right]}{s^3 + 12s^2 + 47s + 84}$$

$$= \frac{\infty}{84}$$

$$= \infty$$

7. เมื่อสัญญาณอินพุท แบบพาราโบลา

$$c_2 = \lim_{s \to 0} \frac{T_E(s)}{s^2}$$

$$\therefore \quad c_2 = \lim_{s \to 0} \frac{1}{s^2} \left[\frac{s^3 + 15s^2 + 59s + 54}{s^3 + 12s^2 + 47s + 84} \right]$$

$$c_2 = \lim_{s \to 0} \frac{1}{\cancel{s^2}} \left[\frac{\frac{1}{\cancel{s^2}} \left(s + 15 + \frac{59}{s} + \frac{54}{s^2} \right)}{s^3 + 12^2 + 47s + 84} \right]$$

$$c_2 = \frac{\infty}{84} = \infty$$

7 จงเขียนกราฟแผนภาพโบด

$$G(s) = \frac{1000(s+5)}{s(s+4)(s^2+30s+1000)}$$

จาก function ทั่วไปของโบด

$$G(j\omega) = \frac{K_a \prod_{i=1}^{R}(1+j\omega T_i)}{(j\omega)^N \prod_{m=1}^{M}(1+j\omega T_m) \prod_{k=1}^{L}\left[\left(1+(2\zeta_k/\omega_{nk})j\omega+(j\omega/\omega_{nk})^2\right)\right]}$$

มี zero = R ตัว

pole ที่จุดกำเนิด = N ตัว

pole อันดับหนึ่ง = M ตัว

pole เชิงซ้อน = L ตัว

ซึ่งมีอยู่ 4 รูปแบบ

1. ค่าอัตราการขยายคงที่ K_a
2. pole หรือ zero ที่จุดกำเนิด $(j\omega)$
3. pole หรือ zero อันดับหนึ่ง $(1+j\omega T)$
4. pole หรือ zero เชิงซ้อน $\left[1+(2\zeta/\omega_n)j\omega+(j\omega/\omega_n)^2\right]$

จากรูปแบบที่ 2

$\cancel{\frac{C(s)}{R(s)}}$

$$G(j\omega) = \frac{1}{1+j\left(\frac{2\zeta\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2}$$

จากโจทย์

$$G(j\omega) = \frac{1000(j\omega+5)}{(j\omega)(j\omega+4)(1000+30(j\omega)+(j\omega)^2)}$$

พิจารณาผลตอบของระบบเมื่อ Input เป็นแบบ Unit step.

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

แทน $s \rightarrow j\omega$

$$G(j\omega) = \frac{\omega_n^2}{(j\omega)^2 + 2\zeta\omega_n(j\omega) + \omega_n^2}$$

หาร ω_n^2 ตลอด

$$G(j\omega) = \frac{1}{\left(j\frac{\omega}{\omega_n}\right)^2 + j2\zeta\frac{\omega}{\omega_n} + 1}$$

$$= \frac{1}{1 + j\left(2\zeta\frac{\omega}{\omega_n}\right) + \left(j\frac{\omega}{\omega_n}\right)^2}$$

เทียบ form

$$\omega_n^2 = 1000$$

$$\omega_n = \sqrt{1000} = 31.62$$

$$2\zeta\omega_n = 30$$

$$2\zeta(31.62) = 30$$

$$\zeta = \frac{30}{2\times(31.62)} = 0.4743$$

③

a)

$$G(j\omega) = \frac{10^3 (5 + j\omega)}{(-\omega^2 + j4\omega)(1000 + j30\omega - \omega^2)}$$

$$G(j\omega) = \frac{10^3 (5 + j\omega)}{(-\omega^2 + j4\omega)([1000 - \omega^2] + j30\omega)}$$

$\cancel{G(j\omega)}$

$$20 \log |G| = 20 \log 10^3 + 20 \log |5 + j\omega|$$

$$- 20 \log |-\omega^2 + j4\omega| - 20 \log \left|[(1000 - \omega^2) + j30\omega]\right|$$

$$20 \log |G| = 60\, dB + 20 \log \sqrt{5^2 + \omega^2} - 20 \log \sqrt{(-\omega^2)^2 + (4\omega)^2}$$

$$- 20 \log \sqrt{(1000 - \omega^2)^2 + (30\omega)^2}$$

$$= 60\, dB + 20 \log \sqrt{5^2 + \omega^2} - 20 \log \sqrt{\omega^4 + 16\omega^2}$$

$$- 20 \log \sqrt{(1000 - \omega^2)^2 + 900\omega^2}$$

at $\omega = 0.1$

$$(dB) = 60\, dB + 20 \log \sqrt{25 + (0.1)^2} - 20 \log \sqrt{(0.1)^4 + 16 (0.1)^2}$$

$$- 20 \log \sqrt{(1000 - (0.1)^2)^2 + 900 (0.1)^2}$$

$$20\log|G| = 60\,dB + 20\log 5 - 20\log 0.4 - 20\log 999.99$$

ที่ $\omega = 0.1$ $=$ 21.97 dB

ที่ $\omega = 10$

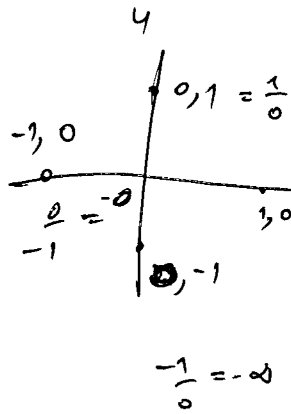
$$20\log|G| = 60\,dB + 20\log\sqrt{25+(10)^2} - 20\log\sqrt{(10)^4+16(10)^2}$$

$$-20\log\sqrt{(1000-(10)^2)^2 + 900(10)^2}$$

$$= 60\,dB + 20\log 11.18 - 20\log 107.7 - 20\log 948.68$$

$$= 60\,dB + 20.96 - 40.64 - 59.54$$

$$= -19.22\ dB$$



$$\phi(\omega) = \tan^{-1}\frac{0}{1000} = 0^\circ$$

$$\boxed{\phi(\omega) = 0^\circ + \tan^{-1}\left(\frac{\omega}{5}\right) - \tan^{-1}\left(\frac{4\omega}{-\omega^2}\right) - \tan^{-1}\left[\frac{30\omega}{(1000-\omega^2)}\right]}$$

$\omega = 0.1$

$$\phi(\omega) = 0^\circ + \tan^{-1}\left(\frac{0.1}{5}\right) - \tan^{-1}\left(\frac{4(0.1)}{-(0.1)^2}\right)$$

$$- \tan^{-1}\left[\frac{30(0.1)}{(1000-(0.1)^2)}\right]$$

$$\phi(\omega) = 0^\circ + 1.145^\circ + 88.56 - 0.1718^\circ$$

$$\phi(\omega) = 89.89^\circ$$

$\omega = 10$

$$\phi(\omega) = 0^\circ + \tan^{-1}\left(\frac{10}{5}\right) - \tan^{-1}\left(\frac{4(10)}{-(10^2)}\right)$$

$$- \tan^{-1}\left[\frac{30(10)}{(1000-(10^2))}\right]$$

$$\phi(\omega) = 0^\circ + 63.43^\circ + 21.8 - 18.43^\circ$$

$$= 66.8^\circ$$