(ແບບຝຶກຫັດ)

(1)

<u>ເສລຍປັນຫາ</u>

Math 3

(1)

$$e^{x+1}\tan y\,dx + \cos y\,dy = 0$$

$$e^{x+1}dx + \frac{\cos y}{\tan y}dy = 0$$

$$e^{x+1}dx + \cos y\,\frac{1}{\frac{\sin y}{\cos y}}\,dy = 0$$

$$e^{x+1}dx + \cos y\,\frac{\cos y}{\sin y}\,dy = 0$$

$$e^{x+1}dx + \cos^2 y\,\operatorname{cosec} y\,dy = 0$$

$$e^{x+1}dx + \left[\tfrac{1}{2}(1+\cos 2y)\right]\operatorname{cosec} y\,dy = 0$$

$$e^{x+1}dx + \tfrac{1}{2}\operatorname{cosec} y\,dy + \tfrac{1}{2}\cos 2y\,\operatorname{cosec} y\,dy = 0$$

$$\cos 2y = 1-2\sin^2 y$$

$$e^{x+1}dx + \tfrac{1}{2}\operatorname{cosec} y\,dy + \tfrac{1}{2}\left[1-2\sin^2 y\right]\frac{1}{\sin y}\,dy = 0$$

$$e^{x+1}dx + \tfrac{1}{2}\operatorname{cosec} y\,dy + \tfrac{1}{2}\left[\frac{1}{\sin y} - \frac{2\sin^2 y}{\sin y}\right]dy = 0$$

$$e^{x+1}d(x+1) + \tfrac{1}{2}\operatorname{cosec} y\,dy + \tfrac{1}{2}\operatorname{cosec} y\,dy - \sin y\,dy = 0$$

$$\int e^{x+1}d(x+1) + \int \operatorname{cosec} y\,dy - \int \sin y\,dy = 0$$

$$e^{x+1} + \ln\left|\operatorname{cosec} y - \cot y\right| + \cos y = C$$

② $\left(y\cos\frac{y}{x} - 2x\sin\frac{y}{x}\right)dx + \left(y - x\cos\frac{y}{x}\right)dy = 0$

แทน $y = vx$

$dy = v\,dx + x\,dv$

$$\left[vx\cos\frac{vx}{x} - 2x\sin\frac{vx}{x}\right]dx + \left(vx - x\cos\frac{vx}{x}\right)(v\,dx + x\,dv) = 0$$

$$\left[vx\cos v - 2x\sin v\right]dx + (vx - x\cos v)(v\,dx + x\,dv) = 0$$

$$x\left[v\cos v - 2\sin v\right]dx + x(v - \cos v)(v\,dx + x\,dv) = 0$$

x หาร ตลอด

$$\left[v\cos v - 2\sin v\right]dx + (v - \cos v)v\,dx + (v - \cos v)x\,dv = 0$$

$$\left[\cancel{v\cos v} - 2\sin v + v^2 - \cancel{v\cos v}\right]dx + (v - \cos v)x\,dv = 0$$

$$\left[v^2 - 2\sin v\right]dx + \left[v - \cos v\right]x\,dv = 0$$

$$\frac{1}{x}\,dx + \frac{[v - \cos v]}{[v^2 - 2\sin v]}\,dv = 0 \quad \text{——— (1)}$$

แทน $x = vy$ $\qquad dx = v\,dy + y\,dv$

$$\left(y\cos\frac{y}{vy} - 2vy\sin\frac{y}{vy}\right)(v\,dy + y\,dv) + \left(y - vy\cos\frac{y}{vy}\right)dy = 0$$

$$\left(y\cos\frac{1}{v} - 2vy\sin\frac{1}{v}\right)(v\,dy + y\,dv) + \left(y - vy\cos\frac{1}{v}\right)dy = 0$$

ดึงตัวร่วม y แล้ว y หารตลอด

$$\left(\cos\frac{1}{v} - 2v\sin\frac{1}{v}\right)(v\,dy + y\,dv) + \left(1 - v\cos\frac{1}{v}\right)dy = 0$$

$$\left(\cos\frac{1}{v} - 2v\sin\frac{1}{v}\right)v\,dy + \left(\cos\frac{1}{v} - 2v\sin\frac{1}{v}\right)y\,dv + \left(1 - v\cos\frac{1}{v}\right)dy = 0$$

$$\left[\cancel{v\cos\frac{1}{v}} - 2v^2\sin\frac{1}{v} + 1 - \cancel{v\cos\frac{1}{v}}\right]dy + \left(\cos\frac{1}{v} - 2v\sin\frac{1}{v}\right)y\,dv = 0$$

$$\left(1 - 2v^2\sin\frac{1}{v}\right)dy + \left(\cos\frac{1}{v} - 2v\sin\frac{1}{v}\right)y\,dv = 0$$

$$\frac{1}{y}dy + \frac{\left(\cos\frac{1}{v} - 2v\sin\frac{1}{v}\right)dv}{\left(1 - 2v^2\sin\frac{1}{v}\right)} = 0 \qquad (2)$$

จาก (1)

$$d(v^2 - 2\sin v) = 2v\,dv - 2\cos v\,dv$$

$$= (2v - 2\cos v)\,dv$$

$$= 2(v - \cos v)\,dv$$

$$\frac{1}{2}d(v^2 - 2\sin v) = (v - \cos v)\,dv \qquad (3)$$

แทน (3) ใน (1)

$$\frac{1}{x}dx + \frac{1}{(v^2 - 2\sin v)}\frac{1}{2}d(v^2 - 2\sin v) = 0$$

$$\int \frac{1}{x}dx + \frac{1}{2}\int \frac{1}{(v^2 - 2\sin v)}d(v^2 - 2\sin v) = 0$$

$$\ln x + \frac{1}{2}\ln(v^2 - 2\sin v) = \ln c$$

$$\ln x + \ln(v^2 - 2\sin v)^{\frac{1}{2}} = \ln c$$

$$\ln\left[x(v^2 - 2\sin v)^{\frac{1}{2}}\right] = \ln c$$

ใส่ e.

$$x(v^2 - 2\sin v)^{\frac{1}{2}} = c$$

ยกกำลังสองทั้งสองข้าง

$$x^2(v^2 - 2\sin v) = c^2 = c'$$

$$x^2v^2 - 2x^2\sin v = c \quad \text{———} \quad ๔$$

จาก $\quad y = vx \quad$ ———

$$v = \frac{y}{x} \quad \text{———} \quad ๕$$

$$v^2 = \frac{y^2}{x^2} \quad \text{———} \quad ๖$$

แทน ๕ ๖ ลงใน ๔

$$\cancel{x^2}\,\frac{y^2}{\cancel{x^2}} - 2x^2\sin\frac{y}{x} = c$$

$$y^2 - 2x^2\sin\frac{y}{x} = c \quad \text{———}$$

Exact

③ $(2xye^{x^2y} + y^2e^{xy^2} + 1)dx + (x^2e^{x^2y} + 2xye^{xy^2} - 2y)dy = 0$

[illegible] පද

$$\underbrace{2xye^{x^2y}dx}_{①} + \underbrace{y^2e^{xy^2}dx}_{③} + dx + \underbrace{x^2e^{x^2y}dy}_{②} + \underbrace{2xye^{xy^2}dy}_{④} - 2ydy = 0$$

පද ① ② $= e^{x^2y}d(x^2y) = e^{x^2y}[x^2dy + ydx^2]$

$$= e^{x^2y}(x^2dy + 2yxdx)$$

$$= \underbrace{x^2e^{x^2y}dy}_{②} + \underbrace{2yxe^{x^2y}dx}_{①} \quad \text{——} \; ⓐ$$

පද ③ ④ $e^{xy^2}dxy^2 = e^{xy^2}[xdy^2 + y^2dx]$

$$= e^{xy^2}[2yxdy + y^2dx]$$

$$= \underbrace{2yxe^{xy^2}dy}_{④} + \underbrace{y^2e^{xy^2}dx}_{③}$$

$$\int dx + \int e^{x^2y}dx^2y + \int e^{xy^2}dxy^2 - \int 2ydy = 0$$

$$x + e^{x^2y} + e^{xy^2} - y^2 = c \quad \text{——} \; ⓑ$$

$$xe^{x^2y}[2xy+2] + ye^{xy^2}(2xy+2) = (2xy+2)(xe^{x^2y} + ye^{xy^2})$$

$$\frac{\partial}{\partial y}[2xye^{x^2y} + y^2e^{xy^2} + 1] = 2x\left[y\frac{\partial}{\partial y}e^{x^2y} + e^{x^2y}\frac{\partial y}{\partial y}\right] + \left[y^2\frac{\partial}{\partial y}e^{xy^2} + e^{xy^2}\frac{\partial y^2}{\partial y}\right] + \frac{\partial 1}{\partial y}$$

$$= 2x\left[ye^{x^2y}x^2 + e^{x^2y}\right] + \left[y^2e^{xy^2}x2y + e^{xy^2}2y\right]$$

$$= 2xyx^2e^{x^2y} + 2xe^{x^2y} + 2xy^3e^{xy^2} + 2ye^{xy^2}$$

(4) หาค่า ปก ของ อินทิเกรต.

$$(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$$

เทียบ $\quad M(x,y)\,dx + N(x,y)\,dy = 0$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy^4e^y + 2xy^3 + y)$$

$$= 2x\left(y^4\frac{\partial e^y}{\partial y} + e^y\frac{\partial y^4}{\partial y}\right) + 2x\frac{\partial y^3}{\partial y} + \frac{\partial y}{\partial y}$$

$$= 2x(y^4e^y + e^y 4y^3) + 6xy^2 + 1$$

$$= 2xy^4e^y + 8xe^yy^3 + 6xy^2 + 1$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^2y^4e^y - x^2y^2 - 3x)$$

$$= 2xy^4e^y - 2xy^2 - 3$$

มิใช่ Exact $\quad \dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$

หาค่า ป.ก ของ อินทิเกรท.

$$\frac{\dfrac{\partial M}{\partial y} - \dfrac{\partial N}{\partial x}}{N(x,y)} = f(x)$$

$$\frac{(\cancel{2xy^4e^y} + 8xe^yy^3 + 6xy^2 + 1 - \cancel{2xy^4e^y} + 2xy^2 + 3)}{x^2y^4e^y - x^2y^2 - 3x}$$

$$\frac{8xy^3e^y + 8xy^2 + 4}{x^2y^4e^y - x^2y^2 - 3x} \qquad (1)$$

$$\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M(x,y)} = g(y)$$

$$\frac{\cancel{2xy^4e^y} - 2xy^2 - 3 - \cancel{2xy^4e^y} - 8xe^y y^3 - 6xy^2 - 1}{2xy^4e^y + 8xe^y y^3 + 6xy^2 + 1}$$

$$\frac{-8xe^y y^3 - 8xy^2 - 4}{2xy^4e^y + 8xe^y y^3 + 6xy^2 + 1} \quad \text{———} \ (3)$$

กรณี ④ $\quad y\cdot f(xy)\,dx + x\cdot g(x,y)\,dy = 0$

และ $f(x,y) \neq g(x,y)$

ตัวประกอบอินทิเกรต $\quad \dfrac{1}{xy(f(xy)-g(xy))}$

จากโจทย์

$$y\underbrace{(2xy^3e^y + 2xy^2 + 1)}_{f(xy)}dx + x\cdot\underbrace{(xy^4e^y - xy^2 - 3)}_{g(xy)}dy = 0$$

$$\frac{1}{xy(2xy^3e^y + 2xy^2 + 1 - xy^4e^y + xy^2 + 3)}$$

$$\frac{1}{xy(2xy^3e^y + 3xy^2 - xy^4e^y + 4)}$$

$$I.F. = \frac{1}{[2x^2y^4e^y + 3x^2y^3 - x^2y^5e^y + 4xy]}$$

เพราะ

$$I.F. = \frac{1}{xy[f(xy) - g(xy)]}$$

$$= \frac{1}{xy[2xy^3e^y + 2xy^2 + 1 - xy^4e^y + xy^2 + 3]}$$

$$= \frac{1}{yx[2xy^3e^y - xy^4e^y + 3xy^2 + 4]}$$

ได้ I.F. คูณตลอด

$$\frac{(2xy^4e^y + 2xy^3 + y)}{yx[2xy^3e^y - xy^4e^y + 3xy^2 + 4]}$$

$$d\,x^2e^y = x^2\,de^y + e^y\,dx^2 = x^2e^y\,dy + 2xe^y\,dx$$

$$d\frac{x^2}{y} = \frac{y\,dx^2 - x^2\,dy}{y^2} = \frac{y\,2x\,dx}{y^2} - \frac{x^2\,dy}{y^2}$$

$$= \frac{2x\,dx}{y} - \frac{x^2\,dy}{y^2}$$

$$d\frac{x}{y^3} = \frac{y^3\,dx - x\,dy^3}{y^6} = \frac{y^3}{y^6}\,dx - \frac{x}{y^6}\,dy^3$$

$$= \frac{1}{y^3}\,dx - \frac{x}{y^6}\,3y^2\,dy$$

$$= \frac{1}{y^3}\,dx - \frac{3x\,dy}{y^4}$$

$$\frac{2xy^4e^y\,dx + x^2y^4e^y\,dy}{y^4}$$

$$\text{I.F.} = \frac{1}{y^4}$$

(ග) $\frac{1}{y^4}$ ගුණකරමු

$$\frac{1}{y^4}(2xy^4e^y + 2xy^3 + y)\,dx + \frac{1}{y^4}(x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$$

$$2xe^y\,dx + \frac{2xy^3\,dx}{y^4} + \frac{y\,dx}{y^4} + x^2e^y\,dy - \frac{x^2y^2\,dy}{y^4} - \frac{3x\,dy}{y^4} =$$

$d\,e^y x^2$

$$\underline{2xe^y\,dx + x^2e^y\,dy} + \underline{\frac{2x\,dx}{y} - \frac{x^2\,dy}{y^2}} + \underline{\frac{1}{y^3}dx - \frac{3x\,dy}{y^4}}$$

$$d\,x^2e^y + d\,\frac{x^2}{y} + d\,\frac{x}{y^3} = 0$$

$$\int d\,x^2e^y + \int d\,\frac{x^2}{y} + \int d\,\frac{x}{y^3} = \int 0$$

$$x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} = c$$

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$$x^3 \frac{dy}{dx} + (2-3x^2)y = x^3$$

$$\frac{dy}{dx} + \frac{(2-3x^2)}{x^3}\, y = 1$$

$$\frac{dy}{dx} + \left(\frac{2}{x^3} - \frac{3}{x}\right) y = 1$$

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$y = e^{-\int P(x)dx}\left[\int Q(x).\, e^{\int P(x)dx}\, dx + c\right]$$

$$y = e^{-\int\left(\frac{2}{x^3}-\frac{3}{x}\right)dx}\left[\int 1.\, e^{\int\left(\frac{2}{x^3}-\frac{3}{x}\right)dx}\, dx + c\right]$$

$$y = e^{-\int\left[2x^{-3}dx - 3\frac{1}{x}dx\right]}\left[\int e^{\int\left[2x^{-3}dx-3\frac{1}{x}dx\right]}\, dx + c\right]$$

$$y = e^{-\left[2\frac{x^{-2}}{-2} - 3\ln x\right]}\left[\int e^{\left[2\frac{x^{-2}}{-2} - 3\ln x\right]}\, dx + c\right]$$

$$y = e^{-\left[-\frac{1}{x^2} - \ln x^3\right]}\left[\int e^{\left[-x^{-2}+\ln x^{-3}\right]}\, dx + c\right]$$

$$y = e^{\left[x^{-2}+\ln x^3\right]}\left[\int e^{\left[-x^{-2}+\ln x^{-3}\right]}\, dx + c\right]$$

$$y = e^{\frac{1}{x^2}}\, \cancel{e}^{\cancel{\ln}\, x^3}\left[\int e^{-\frac{1}{x^2}}\, \cancel{e}^{\cancel{\ln}\, x^{-3}}\, dx + c\right]$$

$$y = e^{\frac{1}{x^2}}\, x^3 \left[\int e^{-\frac{1}{x^2}}\, x^{-3}\, dx + c\right]$$

$$d\left(-\frac{1}{x^2}\right) = -d\,\frac{1}{x^2}$$

$$= -\left[d\,x^{-2}\right]$$

$$= -\left(-2x^{-3}\,dx\right)$$

$$= +2x^{-3}\,dx$$

$$y = e^{\frac{1}{x^2}} x^3 \left[\frac{1}{2}\int e^{-\frac{1}{x^2}}\, d\left(-\frac{1}{x^2}\right) + c\right]$$

$$y = e^{\frac{1}{x^2}} x^3 \left[\frac{1}{2} e^{-\frac{1}{x^2}} + c\right]$$

$$y = \frac{x^3}{2} + e^{\frac{1}{x^2}} x^3 c$$

lo 2 gr apon

$$2y = x^3 + \underbrace{2c}_{c}\, e^{\frac{1}{x^2}} x^3$$

$$2y = x^3 + cx^3 e^{\frac{1}{x^2}}$$

(b) $$2xy' = 10x^3y^5 + y$$

[illegible] $$\frac{dy}{dx} + P(x)y = Q(x)\,y^n$$

$$2x\frac{dy}{dx} - y = 10x^3y^5$$

$$\frac{dy}{dx} - \frac{1}{2x}y = \frac{10x^3}{2x}y^5$$

$$\frac{dy}{dx} - \frac{1}{2x}y = 5x^2y^5 \quad \text{———} \quad (1)$$

$$P(x) = -\frac{1}{2x}$$

$$Q(x) = 5x^2$$

$$y^n = y^5 \longrightarrow n = 5$$

[illegible] $(1) \div y^5$

$$y^{-5}\frac{dy}{dx} - \frac{1}{2x}y^{-4} = 5x^2 \quad \text{———} \quad (2)$$

[illegible] $z = y^{1-5} = y^{-4}$

$$z = y^{-4}$$

$$\frac{dz}{dx} = -4y^{-5}dy$$

$$-\frac{1}{4}\frac{dz}{dx} = y^{-5}dy \quad \text{———} \quad (3)$$

[illegible] (3) [illegible] (2)

$$-\frac{1}{4}\frac{dz}{dx} - \frac{1}{2x}z = 5x^2$$

-4 [illegible]

$$\frac{dz}{dx} + \frac{2}{x} z = -20 x^2$$

$$\frac{dz}{dx} + P(x) z = Q(x)$$

$$P(x) = \frac{2}{x}$$

$$Q(x) = -20 x^2$$

$$z = e^{-\int P(x)\,dx} \left[\int Q(x)\, e^{\int P(x)\,dx}\, dx + c \right]$$

$$z = e^{-\int \frac{2}{x} dx} \left[\int (-20x^2)\, e^{\int \frac{2}{x} dx}\, dx + c \right]$$

แทนค่า

~~หา~~ $e^{-2\int \frac{1}{x} dx} = e^{-2 \ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$

$$e^{\int \frac{2}{x} dx} = e^{2 \ln x} = e^{\ln x^2} = x^2$$

$$z = \frac{1}{x^2} \left[\int (-20 x^2)\, x^2\, dx + c \right]$$

$$z = \frac{1}{x^2} \left[-20 \int x^4\, dx + c \right]$$

$$z = \frac{1}{x^2} \left[-20 \frac{x^5}{5} + c \right] = \frac{1}{x^2} \left[-4x^5 + c \right]$$

$$z = -4 \frac{x^5}{x^2} + \frac{c}{x^2}$$

๔

$$z = -4x^3 + c x^{-2}$$

จาก $z = y^{-4}$

$$y^{-4} = -4x^3 + c x^{-2} \rightarrow \frac{1}{y^4} = -4x^3 + c x^{-2}$$

(7) หาค่า $y = y_c + y_p$

$$(D^2 - 4D + 4)y = 10x^3 e^{2x} + 6x e^{2x}$$

สมการช่วย

$$m^2 - 4m + 4 = 0$$

$$(m-2)(m-2) = 0$$

$m = 2, 2 \longrightarrow$ กรณี $m_1 = m_2$

$$\therefore y_c = c_1 e^{2x} + c_2 x e^{2x}$$

หา y_p

จาก $g(x) = 10x^3 e^{2x} + 6x e^{2x}$

$$y_p = Ax^3 e^{2x} + Bx e^{2x}$$

$$y_p = Ax^5 e^{2x} + Bx^3 e^{2x}$$

$$y'' - 4y' + 4y = 10x^3 e^{2x} + 6x e^{2x} \quad \text{--- (1)}$$

$$y_p' = A\frac{d}{dx}(x^5 e^{2x}) + B\frac{d}{dx}(x^3 e^{2x})$$

$$y_p' = 2Ax^5 e^{2x} + 5Ax^4 e^{2x} + B2x^3 e^{2x} + 3Bx^2 e^{2x}$$

$$y_p' = 2Ax^5 e^{2x} + 5Ax^4 e^{2x} + 2Bx^3 e^{2x} + 3Bx^2 e^{2x}$$

$$y_p'' = 2A\left[x^5\frac{\partial e^{2x}}{\partial x} + e^{2x}\frac{\partial x^5}{\partial x}\right] + 5A\left[x^4\frac{\partial e^{2x}}{\partial x} + e^{2x}\frac{\partial x^4}{\partial x}\right] + 2B\left[x^3\frac{\partial e^{2x}}{\partial x} + e^{2x}\frac{\partial x^3}{\partial x}\right]$$

$$+ 3B\left[x^2\frac{\partial e^{2x}}{\partial x} + e^{2x}\frac{\partial x^2}{\partial x}\right]$$

$$y_p'' = 2A\left[x^5 e^{2x}2 + e^{2x}5x^4\right] + 5A\left[x^4 e^{2x}2 + e^{2x}4x^3\right] + 2B\left[x^3e^{2x}2 + e^{2x}3x^2\right]$$

$$+ 3B\left[x^2e^{2x}2 + e^{2x}2x\right]$$

$$y_p'' = 4Ax^5e^{2x} + 10Ax^4e^{2x} + 10Ax^4e^{2x} + 20Ax^3e^{2x} + 4Bx^3e^{2x} + 6Bx^2e^{2x}$$

$$+ 6Bx^2e^{2x} + 6Bxe^{2x}$$

$$y_p'' = 4Ax^5e^{2x} + 20Ax^4e^{2x} + 20Ax^3e^{2x} + 4Bx^3e^{2x} + 6Bx^2e^{2x} +$$

$$6Bx^2e^{2x} + 6Bxe^{2x}$$

แทน y_p', y_p'' ลงใน ①

$$4Ax^5e^{2x} + 20Ax^4e^{2x} + 20Ax^3e^{2x} + 4Bx^3e^{2x} + 6Bx^2e^{2x} + 6Bx^2e^{2x} + 6Bxe^{2x}$$

$$-8Ax^5e^{2x} - 20Ax^4e^{2x} - 8Bx^3e^{2x} - 12Bx^2e^{2x} + 4Ax^5e^{2x} + 4Bx^3e^{2x}$$

$$\left[\cancel{4Ax^5} + \cancel{20Ax^4} + 20Ax^3 + \cancel{4Bx^3} + \cancel{6Bx^2} + \cancel{6Bx^2} + 6Bx - \cancel{8Ax^5} - \cancel{20Ax^4} - \cancel{8Bx^3}\right.$$

$$\left. - \cancel{12Bx^2} + \cancel{4Ax^5} + \cancel{4Bx^3}\right]e^{2x}$$

$$\left[20Ax^3 + 6Bx\right]e^{2x}$$

$$20Ax^3e^{2x} + 6Bxe^{2x} = 10x^3e^{2x} + 6xe^{2x}$$

$$20A = 10 \longrightarrow A = \frac{10}{20} = \frac{1}{2} \quad \text{——} \quad ②$$

$6\beta = 6$

$\beta = \frac{6}{6} = 1$ —— (9)

$\therefore$

$y_p = \frac{1}{2}x^5 e^{2x} + x^3 e^{2x}$

$\therefore \quad y = y_c + y_p$

$y = c_1 e^{2x} + c_2 x e^{2x} + \frac{1}{2}x^5 e^{2x} + x^3 e^{2x}$ —— (10)