# เฉลย MATH 3

## ปลายภาค 1/2558

ข้อ 1 คำตอบ $-2\sqrt{2}\pi b^2$

หลัก① ใช้สมการอิงตัวแปรเสริม.

$x+y=2b$ → สมการเส้นโค้ง

$x^2+y^2+z^2=2b(x+y)$ → สมการทรงกลม.

$x^2+y^2+z^2=(2b)^2$

รัศมี $=2b$ → รัศมีทรงกลม

$f(x,y,z) = \cancel{ydx+} \; y\hat{i}+z\hat{j}+x\hat{k}$

หลัก②
จากทฤษฎีบทสโตกส์

$$\oint \vec{F}\cdot d\vec{r} = \iint_S (\nabla\times\vec{F})\cdot\hat{n}\,ds$$

$$\oint \vec{F}\cdot d\vec{r} = \oint_C y\,dx+z\,dy+x\,dz$$

$$\vec{F}(x,y,z) = y\hat{i}+z\hat{j}+x\hat{k}$$

$$d\vec{r} = dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}$$

$$\nabla\times\vec{F} = \left|\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & z & x \end{array}\right| \begin{array}{cc} \hat{i} & \hat{j} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ y & z \end{array}$$

$$\nabla \times \vec{F} = \left(\cancelto{0}{\frac{\partial x}{\partial y}} - \cancelto{1}{\frac{\partial z}{\partial z}}\right)\hat{i} + \left(\frac{\partial y}{\partial z} - \cancelto{1}{\frac{\partial x}{\partial x}}\right)\hat{j} + \left(\frac{\partial z}{\partial x} - \cancelto{1}{\frac{\partial y}{\partial y}}\right)\hat{k}$$

$$\nabla \times \vec{F} = -\hat{i} - \hat{j} - \hat{k} \quad \text{———} \quad 1$$

$\hat{n}$ ~~$= \frac{\nabla \vec{F}}{\|\nabla \vec{F}\|}$~~ = vector ปกติ ขนาดหนึ่งหน่วย ของ S

S เป็น ทรงกลม

$$x^2 + y^2 + z^2 - (2b)^2 = 0$$

$$\hat{n} = \frac{\Delta f}{\|\Delta f\|} \quad \text{———} \quad \text{นิยาม } 2.16$$

$$f(x,y,z) = x^2 + y^2 + z^2 - (2b)^2 = 0$$

$$\nabla f(x,y,z) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k}$$

$\nabla f(x,y,z) \Rightarrow$ แยก Diff. ทีละพจน์

$$\frac{\partial f}{\partial x}(x,y,z) = \frac{\partial}{\partial x}\left(x^2 + y^2 + z^2 - (2b)^2\right) = 2x$$

$$\frac{\partial f}{\partial y}(x,y,z) = \frac{\partial}{\partial y}\left(x^2 + y^2 + z^2 - (2b)^2\right) = 2y$$

$$\frac{\partial f}{\partial z}(x,y,z) = \frac{\partial}{\partial z}\left(x^2 + y^2 + z^2 - (2b)^2\right) = 2z$$

$$\hat{n} = \frac{2x\hat{i} + 2y\hat{j} + 2z\hat{k}}{\sqrt{4x^2 + 4y^2 + 4z^2}}$$

$$\hat{n} = \frac{\cancel{2}(x\hat{i} + y\hat{j} + z\hat{k})}{\cancel{2}\sqrt{x^2 + y^2 + z^2}}$$

$$\hat{n} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}} \qquad \text{——— ②}$$

จาก $z = f(x, y)$ → พื้นผิวอยู่บนระนาบ $xy$

$$\therefore \oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, ds \qquad \text{——— ③}$$

แทน ① ② ลงใน ③

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (-\hat{i} - \hat{j} - \hat{k}) \cdot \frac{(x\hat{i} + y\hat{j} + z\hat{k})}{\sqrt{x^2 + y^2 + z^2}}\, ds$$

$$\oint \vec{F} \cdot d\vec{r} = \iint_S \frac{(-x - y - z)}{\sqrt{x^2 + y^2 + z^2}}\, ds \qquad \text{——— ④}$$

จากสมการทรงกลม

$$x^2 + y^2 + z^2 = 2b(x + y) \qquad \text{——— ⑤}$$

และ สมการเส้นโค้ง

$$x^2 + y^2 = 2b \qquad \text{←——— ⑥}$$

Sub (6) into (5)

$$x^2+y^2+z^2 = 2b(2b) = (2b)^2 = 4b^2 \quad \text{—— (9)}$$

Sub (9) into (4)

$$\oint \vec{F}\cdot d\vec{r} = \iint_S \frac{(-x-y-z)}{\sqrt{4b^2}}\, ds$$

$$\oint \vec{F}\cdot d\vec{r} = \iint_S \frac{(-x-y-z)}{2b}\, ds \quad \text{—— (8)}$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{2b} \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\, dA \quad \text{—— (9)}$$

dA from (9)

$$x^2+y^2+z^2 = 4b^2$$

$$z^2 = 4b^2 - x^2 - y^2$$

$$\frac{\partial z^2}{\partial x} = \cancel{\frac{\partial (4b^2)}{\partial x}}^{0} - \frac{\partial x^2}{\partial x} - \cancel{\frac{\partial y^2}{\partial x}}^{0}$$

$$2z\frac{\partial z}{\partial x} = -2x$$

$$\frac{\partial z}{\partial x} = -\frac{2x}{2z} = -\frac{x}{z} \quad \text{—— (10)}$$

$$\frac{\partial z^2}{\partial y} = \cancel{\frac{\partial}{\partial y}(4b^2)}^{0} - \cancel{\frac{\partial x^2}{\partial y}}^{0} - \frac{\partial y^2}{\partial y} = -2y$$

$$2z \frac{\partial z}{\partial y} = -2y$$

$$\frac{\partial z}{\partial y} = -\frac{\cancel{2}y}{\cancel{2}z} = -\frac{y}{z} \quad \text{———} \quad (11)$$

แทน (10) (11) ลงใน (9)

$$\oint \vec{F} \cdot d\vec{r} = \iint_{S_{xy}} \frac{(-x-y-z)}{26} \sqrt{1+(-2x)^2+(-2y)^2} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{26} \sqrt{1+4x^2+4y^2} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{26} \sqrt{1+4(x^2+y^2)} \, dA$$

$$\oint \vec{F} \cdot d\vec{r} = \iint_{S_{xy}} \frac{(-x-y-z)}{26} \sqrt{1+\left(-\frac{x}{z}\right)^2+\left(-\frac{y}{z}\right)^2} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{26} \sqrt{1+\frac{x^2}{z^2}+\frac{y^2}{z^2}} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{26} \sqrt{\frac{z^2}{z^2}+\frac{x^2}{z^2}+\frac{y^2}{z^2}} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{2b} \sqrt{\frac{(\cancel{x} x^2 + y^2 + z^2)}{z^2}} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{2b} \frac{1}{z} \sqrt{(x^2+y^2+z^2)} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{2b} \frac{1}{z} \sqrt{(4b^2)} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{2b} \frac{1}{z} \sqrt{(2b)^2} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{z \cancel{(2b)}} \cancel{(2b)} \, dA$$

$$= \iint_{S_{xy}} \frac{(-x-y-z)}{z} \, dA$$

$$= \iint_{S_{xy}} \frac{-(x+y)-z}{z} \, dA$$

$$= \iint_{S_{xy}} \frac{-(2b)-z}{z} \, dA = \iint_{S_{xy}} \left(-\frac{2b}{z} - 1\right) dA$$

$$= -\iint_{S_{xy}} \frac{2b}{z}\, dA \;\cancel{\phi}\; - \iint_{S_{xy}} dA \quad \text{——— } (12)$$

จาก $x^2 + y^2 + z^2 = (2b)^2 \rightarrow$ ความยาวของรัศมี $= 2b$

รัศมี $r \rightarrow 0 \rightarrow 2b$

$$z^2 = \cancel{4}\,(2b)^2 - x^2 - y^2 \quad \text{——— } (13)$$

$$z = \sqrt{(2b)^2 - x^2 - y^2} \quad \text{——— } (14)$$

เปลี่ยน เป็น พิกัดเชิงขั้ว

$$0 \le r \le 2b$$

$$0 \le \theta \le 2\pi$$

$$x = r\cos\theta \quad \cancel{= 2b\cos\theta}$$

$$y = r\sin\theta \quad \cancel{= 2b\sin\theta}$$

จาก (14)

$$z = \sqrt{(2b)^2 - (r\cos\theta)^2 - (r\sin\theta)^2}$$

$$z = \sqrt{(r)^2 - [r^2\cos^2\theta + r^2\sin^2\theta]}$$

$$z = \sqrt{r^2 - r^2\cancel{(\cos^2\theta + \sin^2\theta)}^{\,1}}$$

$$z = \cancel{\phi}\sqrt{(2b)^2 - r^2}$$

$$= -\int_0^{2\pi}\int_0^{2b} \frac{2b}{\sqrt{(2b)^2 - r^2}}\, r\,dr\,d\theta \;-\; \int_0^{2\pi}\int_0^{2b} r\,dr\,d\theta$$

$$= +b\left[\int_0^{2\pi}\int_0^{2b} \left[(2b)^2 - r^2\right]^{-\frac{1}{2}} d\left[(2b)^2 - r^2\right] \quad d\theta\right] - \int_0^{2\pi} \left[\frac{r^2}{2}\right]\Bigg|_0^{2b} d\theta$$

$$= b\int_0^{2\pi}\left[\frac{\left((2b)^2 - r^2\right)^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_0^{2b}\,\right] d\theta \;-\; \int_0^{2\pi}\left[\frac{(2b)^2}{2}\right] d\theta$$

$$= 2b\int_0^{2\pi}\left[\left[\overset{0}{\cancel{(2b)^2 - (2b)^2}}\right]^{\frac{1}{2}} - \left[(2b)^2 - 0\right]^{\frac{1}{2}}\right] d\theta$$

$$= (2b)^2 \int_0^{2\pi} d\theta$$

$$= (2b)^2\,(2\pi)$$

$$= 8b^2\pi$$

$$= 8b^2\pi$$

$\underline{\underline{Q_o}}$ (i)

ans

$$x = e^{3t}\cos 2t + 3 \quad \text{———} \quad (1)$$

$$\frac{dx}{dt} = \frac{d}{dt}\left(e^{3t}\cos 2t\right) \cancel{\frac{d3}{dt}}^{0}$$

$$" = e^{3t}\frac{d}{dt}\cos 2t + \cos 2t\,\frac{d}{dt}e^{3t}$$

$$" = -e^{3t}\,2\sin 2t + \cos 2t\, e^{3t}\, 3$$

$$\frac{dx}{dt} = \underline{-2e^{3t}\sin 2t + 3e^{3t}\cos 2t} \quad \text{———} \quad (2)$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\left[\text{————————}\right]$$

$$" = -2\frac{d}{dt}\left[e^{3t}\sin 2t\right) + 3\frac{d}{dt}\left[e^{3t}\cos 2t\right]$$

$$" = -2\left[e^{3t}\frac{d}{dt}\sin 2t + \sin 2t\,\frac{d}{dt}e^{3t}\right] + 3\left[e^{3t}\frac{d}{dt}\cos 2t + \cos 2t\,\frac{d}{dt}e^{3t}\right]$$

$$" = -2\left[e^{3t}\cos 2t(2) + \sin 2t\, e^{3t}\, 3\right] + 3\left[e^{3t}(-\sin 2t)2 + \cos 2t\, e^{3t}\, 3\right]$$

$$" = -4e^{3t}\cos 2t - 6e^{3t}\sin 2t - 6e^{3t}\sin 2t + 9e^{3t}\cos 2t$$

$$\frac{d^2x}{dt^2} = 5e^{3t}\cos 2t - 12e^{3t}\sin 2t \quad \text{———} \quad (3)$$

จาก โจทย์

$$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 13x = 39 \quad \text{———} \; ④$$

แทน ① ② ③ ลงใน ④

$$5e^{3t}\cos 2t - 12e^{3t}\sin 2t - 6\left[-2e^{3t}\sin 2t + 3e^{3t}\cos 2t\right] + 13\left[e^{3t}\cos 2t + 3\right] = 39$$

$$\cancel{5e^{3t}\cos 2t} \cancel{- 12e^{3t}\sin 2t} \cancel{+ 12e^{3t}\sin 2t} \cancel{- 18e^{3t}\cos 2t} \cancel{+ 13e^{3t}\cos 2t} + 39 = 39$$

$$39 = 39 \quad \text{———} \; ④$$

แสดงว่า เป็นจริง

მოც. $\cos^2 x \cos 2y = -1$

$y(0) = \frac{\pi}{2}$

№ (7)

$$\sin x \cos 2\cancel{y}\, dx + \cos x \sin 2y \, dy = 0$$

$\cos 2y$ გავყოთ

$$\sin x \, dx + \cos x \frac{\sin 2y}{\cos 2y} dy = 0$$

$\cos x$ გავყოთ

$$\frac{\sin x}{\cos x} dx + \frac{\sin 2y}{\cos 2y} dy = 0$$

$$\int \tan x \, dx + \int \tan 2y \, dy = 0$$

$$\ln \sec |x| + \frac{1}{2} \ln \sec |2y| = \ln c$$

ორ 2 გავამრავლოთ

$$2 \ln \sec |x| + \ln \sec (2y) = 2 \ln c$$

$$\ln \sec^2 |x| + \ln \sec |2y| = \ln c^2$$

$$\ln [\sec^2 x \cdot \sec(2y)] = \ln c^2 = \ln c$$

$$\cancel{e}^{\cancel{\ln}} [\sec^2 x \cdot \sec(2y)] = \cancel{e}^{\cancel{\ln}\, c}$$

$$\sec^2 x \cdot \sec(2y) = c$$

$$\frac{1}{\cos^2 x} \cdot \frac{1}{\cos(2y)} = C$$

$$= C$$

$$\frac{1}{C} = \cancel{\phantom{x}} \quad \cos^2 x \cos(2y)$$

$$\cos^2 x \cos(2y) = C \quad \text{—— (1)}$$

แทนค่า เงื่อนไข

$$y(0) = \frac{\pi}{2}$$

$$\cancel{\overset{1}{\cos^2(0)}} \; \cancel{\overset{-1}{\cos\left(2\cdot\frac{\pi}{2}\right)}} = C$$

$$-1 = C \quad \text{—— (2)}$$

$$\cos^2 x \cos(2y) = -1 \quad \text{——} \; \cancel{\phantom{x}}$$

ข้อ 4

$$2y e^{\frac{x}{y}} dx + (y - 2x e^{\frac{x}{y}}) dy = 0 \quad \text{——— (1)}$$

แทน $x = vy$ ——— (2)

$$dx = v dy + y dv \quad \text{——— (3)}$$

แทน (2) (3) ลงใน (1)

$$2y e^{\left(\frac{vy}{y}\right)} (v dy + y dv) + (y - 2(vy) e^{\frac{vy}{y}}) dy = 0$$

$$2y e^{v} (v dy + y dv) + (y - 2vy e^{v}) dy = 0$$

$$2y e^{v} v dy + 2y^2 e^{v} dv + y dy - 2vy e^{v} dy = 0$$

$$\cancel{2vy e^{v} dy} + 2y^2 e^{v} dv + y dy - \cancel{2vy e^{v} dy} = 0$$

$$2y^2 e^{v} dv + y dy = 0$$

$y^2$ หารตลอด

$$2\int e^{v} dv + \int \frac{1}{y} dy = \int 0$$

$$2e^{v} + \ln|y| = C \quad \text{——— (4)}$$

แทน $x = vy$

$$v = \frac{x}{y} \quad \text{——— (5)} \quad \#$$

แทน (4) ใน (5)

$$2e^{\frac{x}{y}} + \ln|y| = C \quad \text{———} \#$$

ข้อ 5

$$(y^2 e^{xy^2} + 4x^3)\,dx + (2xy\, e^{xy^2} - 3y^2)\,dy = 0$$

$\cancel{y^2 e^{xy^2}}$ รูปแบบ สมการ

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$M(x,y) = y^2 e^{xy^2} + 4x^3$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(y^2 e^{xy^2}) + \cancel{\frac{\partial}{\partial y}(4x^3)}^{\,0}$$

$$= y^2 \frac{\partial}{\partial y} e^{xy^2} + e^{xy^2} \frac{\partial}{\partial y} y^2$$

$$= y^2 e^{xy^2}(x2y) + 2y\, e^{xy^2}$$

$$\frac{\partial M}{\partial y} = 2y^3 x e^{xy^2} + 2y\, e^{xy^2} \quad (1)$$

$$N(x,y) = 2xy\, e^{xy^2} - 3y^2$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(2xy\, e^{xy^2}) - \cancel{\frac{\partial}{\partial x}(3y^2)}^{\,0}$$

$$= \cancel{2\frac{\partial}{\partial x}}\; 2y\left[\frac{\partial}{\partial x}(x\, e^{xy^2})\right]$$

$$= 2y\left[x \frac{\partial}{\partial x} e^{xy^2} + e^{xy^2} \cancel{\frac{\partial x}{\partial x}}^{\,1}\right]$$

$$\frac{\partial N}{\partial x} = 2y\left[x e^{xy^2} y^2 + e^{xy^2}\right]$$

$$\frac{\partial N}{\partial x} = 2y^3 x e^{xy^2} + 2y e^{xy^2} \quad \text{———} \quad (2)$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$ ⟶ เป็น สมการ Exact

จากโจทย์ แยกพจน์ จะจับ เทอม

$$\underline{y^2 e^{xy^2} dx} + 4x^3 dx + \underline{2xy e^{xy^2} dy} - 3y^2 dy = 0$$

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$$d\, e^{xy^2} x = e^{xy^2} dx + x\, d e^{xy^2}$$

$$= e^{xy^2} dx + x e^{xy^2} d xy^2$$

✗

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$$d\, e^{xy^2} y^2 = e^{xy^2} dy^2 + y^2 d e^{xy^2}$$

$$= 2y e^{xy^2} dy + y^2 \left[e^{xy^2} d xy^2\right)$$

$$= 2y e^{xy^2} dy + y^2 \left[e^{xy^2} (x\, dy^2 + y^2 dx)\right])$$

$$= 2y e^{xy^2} dy + y^2 e^{xy^2} \left[2xy\, dy + y^2 dx\right)$$

$$= 2y e^{xy^2} dy + 2xy^3 e^{xy^2} dy + y^4 e^{xy^2} dx$$ — e

$$d\,e^{xy^2} = e^{xy^2}\left[d\,xy^2\right]$$

$$= e^{xy^2}\left[x\,dy^2 + y^2\,dx\right]$$

$$= e^{xy^2}\left[x\,2y\,dy + y^2\,dx\right]$$

$$= 2yx\,e^{xy^2}\,dy + y^2 e^{xy^2}\,dx$$

จากโจทย์

$$\int d\,e^{xy^2} + 4\int x^3\,dx - 3\int y^2\,dy = 0$$

$$e^{xy^2} + \cancel{4}\frac{x^4}{\cancel{4}} - \cancel{3}\frac{y^3}{\cancel{3}} = C$$

$$e^{xy^2} + x^4 - y^3 = C$$

แทนค่าเริ่มต้น

$$y(1) = 0$$

$$e^{(1)(0)^2} + (1)^4 - (0)^3 = C$$

$$1 + 1 - 0 = C$$

$$2 = C$$

$$e^{xy^2} + x^4 - y^3 = 2$$

ตอบ $x^3y^4 - 4y^3 = c$

ข้อ 6

$$\underbrace{3x^2y^2}_{M(x,y)}\,dx + \underbrace{4(x^3y-3)}_{N(x,y)}\,dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(3x^2y^2) = 3x^2\frac{\partial y^2}{\partial y} = 6x^2y$$

$$\frac{\partial M}{\partial y} = 6yx^2$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(4(x^3y-3)\right)$$

$$= 4y\frac{\partial x^3}{\partial x} + \cancel{2\frac{(-12)}{\partial x}}^{0}$$

$$12yx^2$$

ทดสอบ กรณี 1

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N(x,y)} = f(x)$$

$$\frac{6yx^2 - 12yx^2}{4(x^3y-3)} = \frac{-6yx^2}{4(x^3y-3)} \neq f(x)$$

ทดสอบ กรณี 2

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M(x,y)} = g(y)$$

$$\frac{12yx^2 - 6yx^2}{3x^2y^2} = \frac{6yx^2}{3x^2y^2} = \frac{2}{y} = g(y)$$

ใช้ได้

หาตัวประกอบอินทิเกรต (I.F)

$$I.F. = e^{\int g(y)\,dy} = e^{\int \frac{2}{y}\,dy}$$

$$= e^{2\int \frac{1}{y}\,dy}$$

$$= e^{2\ln|y|}$$

$$= e^{\ln|y|^2}$$

$$I.F. = y^2$$

นำ I.F. คูณตลอด

$$(y^2)\,3x^2y^2\,dx + 4(y)^2(x^3y-3)\,dy = 0$$

$$\underline{3x^2y^4\,dx} + \underline{4x^3y^3\,dy} - 12y^2\,dy = 0$$

$$\rightarrow d\,x^3y^4 = x^3\,dy^4 + y^4\,dx^3$$

$$= 4y^3x^3\,dy + 3y^4x^2\,dx$$

$\therefore$ ทำ Int

$$\int d\,x^3y^4 - 12\int y^2\,dy = \int 0$$

$$x^3y^4 - 12\frac{y^3}{3} = C$$

$$x^3y^4 - 4y^3 = C$$

$$y = (\tan^{-1} x + c](x^2+1)$$

ข้อ 9

$$\frac{dy}{dx} - \frac{2x}{(x^2-1)} y = 1$$

อยู่ใน รูปแบบ ข้างล่าง

$$\frac{dy}{dx} + P(x) y = Q(x)$$

$$P(x) = -\frac{2x}{(x^2-1)}$$

$$Q(x) = 1$$

$$y = e^{-\int P(x)\,dx}\left[\int Q(x)\, e^{\int P(x)\,dx}\, dx + c\right]$$

$$y = e^{-\int \left(-\frac{2x}{(x^2-1)}\right) dx}\left[\int (1)\, e^{\int \left(\frac{-2x}{(x^2-1)}\right) dx}\, dx + c\right]$$

$$y = e^{\int \frac{1}{(x^2-1)} d(x^2-1)}\left[\int e^{-\int \frac{1}{(x^2-1)} d(x^2-1)}\, dx + c\right]$$

$$y = \cancel{e}^{\cancel{\ln}(x^2-1)}\left[\int e^{-\ln(x^2-1)}\, dx + c\right]$$

$$y = (x^2-1)\left[\int \cancel{e}^{\cancel{\ln}(x^2-1)^{-1}}\, dx + c\right]$$

$$y = (x^2-1)\left[\int \frac{1}{(x^2-1)}\, dx + c\right]$$

$$\boxed{\int \frac{du}{u^2-a^2} = \frac{1}{2a} \ln\left(\frac{u-a}{u+a}\right) + c}$$

$$y = (x^2-1)\left[\frac{1}{2(1)} \ln \frac{(x-1)}{(x+1)} + c\right)$$

$$y = \frac{(x^2-1)}{2} \ln \frac{(x-1)}{(x+1)} + (x^2-1)\, c$$

ข้อ 8

$$x\frac{dy}{dx} + y + 2x^6y^4 = 0$$

จาก รูปแบบ สมการ[illegible]

$$\frac{dy}{dx} + p(x)y = Q(x)y^n$$

จากโจทย์ เอา $x$ หารตลอด

$$\frac{\cancel{x}}{\cancel{x}}\frac{dy}{dx} + \frac{y}{x} + 2\frac{x^6}{x}y^4 = 0$$

$$\frac{dy}{dx} + \frac{y}{x} = -2x^5y^4$$

$$\frac{dy}{dx} + \frac{1}{x}y = -2x^5y^4 \quad \text{——— } ①$$

$$p(x) = \frac{1}{x}$$

$$Q(x) = -2x^5$$

$$y^n = y^4 \text{ ——o } n = 4$$

เอา $y^4$ หาร ตลอด ①

$$\frac{1}{y^4}\frac{dy}{dx} + \frac{1}{x}\frac{y}{y^4} = -2x^5\frac{\cancel{y^4}}{\cancel{y^4}}$$

$$y^{-4}\frac{dy}{dx} + \frac{1}{x}y^{-3} = -2x^5 \quad \text{——— } ②$$

ให้ $z = y^{-3}$

$$\frac{dz}{dx} = -3y^{-4}\frac{dy}{dx}$$

$$-\frac{1}{3}\frac{dz}{dx} = y^{-4}\frac{dy}{dx} \quad \text{———} \quad ③$$

ඉන ③ ආදේශ ව ②

$$-\frac{1}{3}\frac{dz}{dx} + \frac{1}{x}z = -2x^5$$

ශ $-3$ ගුණ කරමු

$$\frac{dz}{dx} - \frac{3}{x}z = +6x^5 \quad \text{———} \quad ④$$

එය දැන් linear.

$$z = e^{-\int P(x)\,dx}\left[\int Q(x)\,e^{\int P(x)\,dx}\,dx + C\right]$$

$$P(x) = -\frac{3}{x}$$

$$Q(x) = 6x^5$$

$$z = e^{-\int(-\frac{3}{x})\,dx}\left[\int (6x^5)\,e^{\int(-\frac{3}{x})\,dx}\,dx + C\right]$$

$$z = e^{3\int\frac{1}{x}\,dx}\left[6\int x^5\,e^{-3\int\frac{1}{x}\,dx}\,dx + C\right]$$

$$z = e^{3\ln x}\left[6\int x^5\,e^{-3\ln x}\,dx + C\right]$$

$$z = \cancel{e^{\ln}}\,x^3\left[6\int x^5\,\cancel{e^{\ln}}\,x^{-3}\,dx + C\right]$$

$$z = x^3 \left[ 6 \int \frac{x^5}{x^3}\, dx + c \right]$$

$$z = \cancel{4} x^3 \left[ 6 \int x^2\, dx + c \right]$$

$$z = x^3 \left[ 6 \frac{x^3}{3} + c \right]$$

$$z = x^3 \left[ 2x^3 + c \right]$$

$$z = 2x^6 + x^3 c \qquad (5)$$

จาก $z = y^{-3} = \dfrac{1}{y^3}$ แทน ลงใน (5)

$$\frac{1}{y^3} = 2x^6 + x^3 c \qquad (6)$$

ข้อ ⑤

$$(D^2 - 4D + 4)y = 10x^3e^{2x} + 6xe^{2x}$$

หา $y_c$ → โดยใช้สมการช่วย.

$$(m^2 - 4m + 4)\,y = 0$$

$$(m^2 - 4m + 4) = 0$$

$$(m-2)(m-2) = 0$$

$$\left.\begin{array}{l} m_1 = 2 \\ m_2 = 2 \end{array}\right\} \text{กรณี } m_1 = m_2$$

$$y_c = c_1e^{2x} + c_2xe^{2x}$$

หา $y_p$ → โดยใช้วิธีเทียบ ส.ป.ส.

สมมุติ $y_p$ จาก $Q(x)$

$$y_p = Ax^3e^{2x} + \underbrace{bxe^{2x}}_{①}$$

พจน์ ① ซ้ำกับ $y_c$ → ∴ ต้องคูณด้วย $x^2$ ตลอด

$$y_p = Ax^5e^{2x} + bx^3e^{2x} \quad ①$$

$$\frac{d^2y}{dx^2} = 2A\left[x^5 e^{2x} 2 + e^{2x} 5x^4\right] + 8A\left[x^4 e^{2x} 2 + e^{2x} 4x^3\right]$$

$$+ 2b\left[x^3 e^{2x} 2 + e^{2x} 3x^2\right] + 3b\left[x^2 e^{2x} 2 + e^{2x} 2x\right]$$

$$\frac{d^2y}{dx^2} = 4Ax^5e^{2x} + 10Ax^4e^{2x} + 10Ax^4e^{2x} + 20Ax^3e^{2x}$$

$$+ 4bx^3e^{2x} + 6bx^2e^{2x} + 6bx^2e^{2x} + 6bxe^{2x}$$

$$\frac{d^2y}{dx^2} = 4Ax^5e^{2x} + (20A)x^4e^{2x} + (20A+4b)x^3e^{2x}$$

$$+ (12b)x^2e^{2x} + 6bxe^{2x} \quad \text{———} \quad (4)$$

แทน (1) (3) (4) ลงใน (2)

$$20Ax^3e^{2x} + \cancel{4bx^3e^{2x}}$$

$$\cancel{4Ax^5e^{2x}} + \cancel{20Ax^4e^{2x}} + \boxed{(20A+4b)x^3e^{2x}} + \cancel{12bx^2e^{2x}} + 6bxe^{2x}$$

$$-\cancel{8Ax^5e^{2x}} - \cancel{20Ax^4e^{2x}} - \cancel{8bx^3e^{2x}} - \cancel{12bx^2e^{2x}} + \cancel{4Ax^5e^{2x}} + \cancel{4bx^3e^{2x}}$$

$$= 10x^3e^{2x} + 6xe^{2x}$$

$$20Ax^3e^{2x} + 6bxe^{2x} = 10x^3e^{2x} + 6xe^{2x}$$

เทียบ ส.ป.ส.

$$20A = 10$$

$$A = \frac{10}{20} = \frac{1}{2}$$

$$6b = 6$$

$$b = \frac{6}{6} = 1$$

$$A = \frac{1}{2}$$

$$b = 1$$

$$y_p = \frac{1}{2}x^5e^{2x} + x^3e^{2x}$$

$$y = y_c + y_p$$

$$y = c_1e^{2x} + c_2xe^{2x} + \frac{1}{2}x^5e^{2x} + x^3e^{2x}$$