Kuas.

Nasima Math 4

1

(1) $$(3x + xy^2)\,dx - (y + x^2y)\,dy = 0$$

$$\cancel{3x\,dx} + xy^2\,dx - (y\,dy + x^2y\,dy) = 0$$

$$\cancel{3x\,dx +} xy^2\,dx - y\,dy - x^2y\,dy = 0$$

$$x(3 + y^2)\,dx - y(1 + x^2)\,dy = 0$$

$$\frac{x\,dx}{(1+x^2)} - \frac{y}{(3+y^2)}\,dy = 0$$

$$\frac{1}{(1+x^2)}\,d(1+x^2) - \frac{1}{(3+y^2)}\,d(3+y^2) = 0$$

$$\frac{1}{2}\int \frac{1}{(1+x^2)}\,d(1+x^2) - \frac{1}{2}\int \frac{1}{(3+y^2)}\,d(3+y^2) = 0$$

$$\frac{1}{2}\ln(1+x^2) - \frac{1}{2}\ln(3+y^2) = \ln c$$

$$\ln(1+x^2)^{\frac{1}{2}} - \ln(3+y^2)^{\frac{1}{2}} = \ln c$$

$$\ln\frac{(1+x^2)^{\frac{1}{2}}}{(3+y^2)^{\frac{1}{2}}} = \ln c$$

$$\ln\left[\frac{(1+x^2)}{(3+y^2)}\right]^{\frac{1}{2}} = \ln c$$

take e

$$\left[\frac{(1+x^2)}{(3+y^2)}\right]^{\frac{1}{2}} = c$$

$$y(1) = 3$$

$$\left[\frac{(1+1^2)}{(3+3^2)}\right]^{\frac{1}{2}} = c$$

$$\sqrt{\frac{2}{12}} = c$$

$$\sqrt{\frac{1}{6}} = c$$

$$\therefore \left[\frac{(1+x^2)}{(3+y^2)}\right]^{\frac{1}{2}} = \sqrt{\frac{1}{6}} \longrightarrow \text{(b)}$$

(2) $(1+2e^{\frac{x}{y}})dx + 2e^{\frac{x}{y}}(1-\frac{x}{y})\,dy = 0$

~~use~~ $\cancel{y = vx \to dy = vdx + xdv}$

$\cancel{(1+2e^{\frac{x}{vx}})dx + 2e}$

use $x = vy \to dx = vdy + ydv$

$(1+2e^{\frac{vy}{y}})(vdy+ydv) + 2e^{\frac{vy}{y}}(1-\frac{vy}{y})\,dy = 0$

$(1+2e^{v})(vdy+ydv) + 2e^{v}(1-v)\,dy = 0$

$vdy + 2e^{v}\cancel{v dy} + ydv + 2e^{v}ydv + 2e^{v}dy - 2e^{v}\cancel{vdy} = 0$

$(v+2e^{v})dy + (1+2e^{v})ydv = 0$

$\frac{1}{y}dy + \frac{(1+2e^{v})}{(v+2e^{v})}dv = 0$

$\frac{1}{(v+2e^{v})}\,d(v+2e^{v}) = \frac{1}{(v+2e^{v})}(dv+2de^{v})$

$= \frac{1}{(\quad)}(dv+2e^{v}dv)$

$\frac{1}{y}dy + \frac{1}{(v+2e^{v})}\,d(v+2e^{v}) = \int 0$

$\ln y + \ln(v+2e^{v}) = \ln c$

$$\ln y(v + 2e^{v}) = \ln c$$

take e

$$y(v + 2e^{v}) = c$$

on $x = vy$

$$v = \frac{x}{y}$$

$$y\left(\frac{x}{y} + 2e^{\frac{x}{y}}\right) = c$$

$$x + 2ye^{\frac{x}{y}} = c$$

(1) $$\frac{xy-1}{x^2y}\,dx - \frac{1}{xy^2}\,dy = 0$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(\frac{xy-1}{x^2y}\right) = \frac{x^2y\frac{\partial}{\partial y}(xy-1) - (xy-1)\frac{\partial}{\partial y}x^2y}{(x^2y)^2}$$

$$= \frac{x^2y\,x - (xy-1)x^2}{x^4y^2}$$

$$= \frac{\cancel{x^3y} - \cancel{x^3y} + x^2}{x^4y^2} = \frac{x^2}{x^4y^2} = \frac{1}{x^2y^2}$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}\left(-\frac{1}{xy^2}\right) = -\frac{\partial}{\partial x}(xy^2)^{-1}$$

$$\frac{\partial N}{\partial x} = -(-1)(xy^2)^{-2}\frac{\partial(xy^2)}{\partial x}$$

$$\frac{\partial N}{\partial x} = (xy^2)^{-2}\,y^2 = \frac{y^2}{(xy^2)^2} = \frac{y^2}{x^2y^4}$$

$$\frac{\partial N}{\partial x} = \frac{1}{x^2y^2} \longrightarrow \text{B}$$

$$u(x,y) = \int^x M(x,y)\,dx + \varphi(y)$$

$$u(x,y) = \int^x \frac{(xy-1)}{x^2y}\,dx + \varphi(y)$$

$$= \int^x \frac{xy}{x^2y}\,dx - \int^x \frac{1}{x^2y}\,dx + \varphi(y)$$

$$u(x,y) = \int^{x} \frac{1}{x}\,dx - \int^{x} \frac{1}{x^2 y}\,dx + p(y)$$

$$u(x,y) = \ln x - \frac{1}{y}\int^{x} x^{-2}\,dx + p(y)$$

$$u(x,y) = \ln x - \frac{1}{y}\,\frac{x^{-2+1}}{-2+1} + p(y)$$

$$u(x,y) = \ln x - \frac{1}{y}\,\frac{x^{-1}}{-1} + p(y)$$

$$u(x,y) = \ln x + \frac{1}{xy} + p(y) \quad \text{———} \quad (1)$$

$$\frac{\partial u(x,y)}{\partial y} = \cancel{\frac{\partial \ln x}{\partial y}}^{0} + \frac{\partial}{\partial y}\frac{1}{xy} + p'(y)$$

$$= \frac{1}{x}\,\frac{\partial}{\partial y}\frac{1}{y} + p'(y)$$

$$= \frac{1}{x}\,\frac{\partial}{\partial y} y^{-1} + p'(y)$$

$$= \frac{1}{x}(-1)\,y^{-2} + p'(y)$$

$$\frac{\partial u(x,y)}{\partial y} = -\frac{1}{xy^2} + p'(y)$$

$$\frac{\partial u(x,y)}{\partial y} = N(x,y)$$

$$-\frac{1}{xy^2} + p'(y) = -\frac{1}{xy^2}$$

$$p'(y) = 0$$

$$p(y) = \int 0\,dy = C \quad \text{———} \quad (2)$$

[illegible] ② in ①

$$U(x,y) = \ln x + \frac{1}{xy} + c = C$$

$$\ln x + \frac{1}{xy} = C - c = C$$

$$\frac{1}{xy} + \ln x = C$$

$$\frac{\partial N}{\partial x} = y^4 e^y 2x - y^2 2x - 3$$

$$= 2x e^y y^4 - 2x y^2 - 3$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N(x,y)} = \frac{\cancel{2x y^4 e^y} + 8x e^y y^3 + 6xy^2 + 1 - \cancel{2x e^y y^4} + 2xy^2 + 3}{x^2 y^4 e^y - x^2 y^2 - 3x}$$

$$= \frac{8x e^y y^3 + 8x y^2 + 4}{x^2 y^4 e^y - x^2 y^2 - 3x} \quad \text{—— } b$$

(9)

$$(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4e^y - x^2y^2 - 3x)\,dy = 0$$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N(x,y)} = f(x) \qquad I.F = e^{\int f(x)dx}$$

$$\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M(x,y)} = g(y) \longrightarrow I.F = e^{\int g(y)dy}$$

$$\frac{\partial M}{\partial y} = \frac{\partial}{\partial y}(2xy^4e^y + 2xy^3 + y)$$

$$= 2x\frac{\partial}{\partial y}(y^4e^y) + 2x\frac{\partial y^3}{\partial y} + \frac{\partial y}{\partial y}$$

$$= 2x\left(y^4\frac{\partial e^y}{\partial y} + e^y\frac{\partial y^4}{\partial y}\right) + 6xy^2\frac{\partial y}{\partial y} + 1$$

$$= 2x(y^4e^y + 4e^y y^3) + 6xy^2 + 1$$

$$\frac{\partial M}{\partial y} = 2xy^4e^y + 8xe^y y^3 + 6xy^2 + 1$$

$$\frac{\partial N}{\partial x} = \frac{\partial}{\partial x}(x^2y^4e^y - x^2y^2 - 3x)$$

$$= y^4e^y\frac{\partial x^2}{\partial x} - y^2\frac{\partial x^2}{\partial x} - 3\frac{\partial x}{\partial x}$$

~~๕~~

ใช้ทฤ ๔ ~~๕~~

$$y\,f(x,y)\,dx + x\,g(x,y)\,dy = 0$$

ตามโจทย์

$$y(2xy^3e^y + 2xy^2 + 1)\,dx + x(xy^4e^y - xy^2 - 3)\,dy = 0$$

$$f(x,y) = 2xy^3e^y + 2xy^2 + 1$$

$$g(x,y) = xy^4e^y - xy^2 - 3$$

$$I.F = \frac{1}{xy(f(x,y) - g(x,y))} = \frac{1}{xy(2xy^3e^y + 2xy^2 + 1 - xy^4e^y + xy^2 + 3)}$$

$$= \frac{1}{2x^2y^4e^y + 2x^2y^3 + xy - x^2y^5e^y + x^2y^3 + 3xy}$$

$$= \frac{1}{2x^2y^4e^y + 3x^2y^3 + 4xy - x^2y^5e^y}$$

ค ดังหรือยาก

$$I.F = \frac{1}{y^4}$$

http://AV101.hot.to

av101.dtiblog.com

ที่ ๔ 5 ๖ เฉลยแล้ว

ข้อ (7) โจทย์ ข้อ

$$\frac{dy}{dx} = -\frac{4}{x^2} - \frac{1}{x}y + y^2 \longrightarrow y_1 = \frac{2}{x} \text{ ——— โจทย์ให้}$$

$$z = \frac{1}{y - y_1} = \frac{1}{y - \frac{2}{x}}$$

$$z\left(y - \frac{2}{x}\right) = 1$$

$$y - \frac{2}{x} = \frac{1}{z}$$

$$y = \frac{1}{z} + \frac{2}{x} \quad \text{———} \quad (1)$$

$$\frac{dy}{dx} = \frac{d z^{-1}}{dx} + \frac{d}{dx} 2x^{-1}$$

$$\frac{dy}{dx} = (-1) z^{-2} \frac{dz}{dx} + 2(-1) x^{-2} \cancel{\frac{dx}{dx}}^{1}$$

$$\frac{dy}{dx} = -\frac{1}{z^2}\frac{dz}{dx} - \frac{2}{x^2} \quad \text{———} \quad (2)$$

(1) และ (2) แทนโจทย์

$$-\frac{1}{z^2}\frac{dz}{dx} - \frac{2}{x^2} = -\frac{4}{x^2} - \frac{1}{x}\left(\frac{1}{z} + \frac{2}{x}\right) + \left(\frac{1}{z} + \frac{2}{x}\right)^2$$

$$-\frac{1}{z^2}\frac{dz}{dx} - \frac{2}{x^2} = \cancel{-\frac{4}{x^2}} - \frac{1}{xz} - \frac{2}{x^2} + \frac{1}{z^2} + \frac{4}{zx} + \cancel{\frac{4}{x^2}}$$

$$-\frac{1}{z^2}\frac{dz}{dx} = \cancel{+\frac{2}{x^2}} - \frac{1}{xz} \cancel{-\frac{2}{x^2}} + \frac{1}{z^2} + \frac{4}{xz}$$

$$-\frac{1}{z^2}\frac{dz}{dx} = -\frac{1}{xz} + \frac{1}{z^2} + \frac{4}{xz}$$

$-z^2$ คูณตลอด

$$\frac{dz}{dx} = +\frac{1}{x} z - 1 - \frac{4}{x} z$$

$$\frac{dz}{dx} = \left(\frac{1}{x} - \frac{4}{x}\right) z - 1$$

$$\frac{dz}{dx} = -\frac{3}{x} z - 1$$

$$\frac{dz}{dx} + \frac{3}{x} z = -1$$

$$P(x) = \frac{3}{x}$$

$$Q(x) = -1$$

$$z = e^{-\int P(x)\,dx}\left[\int e^{\int P(x)\,dx} Q(x)\,dx + C\right]$$

$$z = e^{-\int \frac{3}{x} dx}\left[\int e^{\int \frac{3}{x} dx}(-1)\,dx + C\right]$$

$$z = e^{-3\ln x}\left[\int e^{3\ln x}(-1)\,dx + C\right]$$

$$z = e^{\ln x^{-3}}\left[\int e^{\ln x^{3}}(-1)\,dx + C\right]$$

$$z = \frac{1}{x^3}\left[\int -x^3\,dx + C\right]$$

$$z = \frac{1}{x^3}\left[-\int x^3\,dx + C\right]$$

$$z = \frac{1}{x^3}\left[-\frac{x^4}{4} + C\right] = -\frac{\cancel{x^4}}{\cancel{x^3}\,4} + \frac{C}{x^3}$$

$$z = -\frac{x}{4} + \frac{C}{x^3}$$

[illegible] $z = \dfrac{1}{y - \frac{2}{x}}$

$$\frac{1}{y - \frac{2}{x}} = -\frac{x}{4} + \frac{c}{x^3} = -\left(x/4\right) + c\,x^{-3}$$

$$1 = \left[y - \frac{2}{x}\right]\left[-\frac{x}{4} + \frac{c}{x^3}\right]$$

$$y = \frac{1}{\left[\frac{c}{x^3} - \frac{x}{4}\right]} + \frac{2}{x}$$

(8)

$$\frac{dy}{dx} = \frac{x - 2y + 5}{2x - y + 4}$$

เป็น form $\qquad \dfrac{dy}{dx} = \dfrac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$

$$a_1b_2 - a_2b_1 = (1)(-1) - (2)(-2) = -1 + 4 = 3 \neq 0$$

กำหนดตัวแปรใหม่ $\qquad x = u + h$

$$y = v + k$$

เมื่อ h และ k สอดคล้องกับระบบสมการ

$1h - 2k + 5 = 0$ —— (1)

$2h - k + 4 = 0$ —— (2)

จาก (1) $\qquad -2k = -5 - h$

$$k = \frac{-5-h}{-2} = \frac{-(5+h)}{-2} = \frac{5+h}{2} \quad \text{—— (3)}$$

แทน (3) ใน (2)

$$2h - \left(\frac{5+h}{2}\right) + 4 = 0$$

$$2h - \frac{5}{2} - \frac{h}{2} + 4 = 0$$

$$2h - \frac{h}{2} = -4 + \frac{5}{2}$$

$$\frac{4h - h}{2} = \frac{-8+5}{2}$$

$$\frac{3}{2}h = -\frac{3}{2}$$

$$h = -\frac{3}{2} \times \frac{2}{3} = -1 \quad \text{—— (4)}$$

แทน h ใน (3)

$$k = \frac{5 + (-1)}{2} = \frac{4}{2} = 2 \quad \text{——}$$

$h = -1$

$k = 2$

$x = u - 1 \qquad dx = du$

$y = v + 2 \qquad dy = dv$

$$\frac{dv}{du} = \frac{(u-1) - 2(v+2) + 5}{2(u-1) - (v+2) + 4}$$

$$\frac{dv}{du} = \frac{u - 1 - 2v - 4 + 5}{2u - 2 - v - 2 + 4} = \frac{u - 2v}{2u - v}$$

∴ เป็นสมการเอกพันธ์ ให้ $z = \frac{v}{u} \rightarrow v = zu$ —— ①

$$\frac{dv}{du} = z\frac{du}{du} + u\frac{dz}{du}$$

จาก $\frac{dv}{du} = \frac{u - 2v}{2u - v}$ —— ②

$$\frac{dv}{du} = z + u\frac{dz}{du}$$

$$\frac{dv}{du} = z + u\frac{dz}{du} \text{ —— ③}$$

แทน ③ ① ลงใน ②

$$z + u\frac{dz}{du} = \frac{u - 2zu}{2u - zu} = \frac{\cancel{u}(1 - 2z)}{\cancel{u}(2 - z)}$$

$$z + u\frac{dz}{du} = \frac{(1 - 2z)}{(2 - z)}$$

$$u\frac{dz}{du} = \frac{(1 - 2z)}{(2 - z)} - z = \frac{1 - 2z - (2 - z)z}{(2 - z)}$$

$$u\frac{dz}{du} = \frac{1 - 2z - 2z + z^2}{2 - z} = \frac{z^2 - 4z + 1}{2 - z}$$

$$\frac{(2-z)\,dz}{z^2-4z+1} = \frac{1}{u}\,du$$

$$\cancel{\frac{(2-z)}{z^2-4z+1} = \frac{(2-z)}{2z}}$$

$$d(z^2-4z+1) = 2z\,dz - 4\,dz$$
$$= (2z-4)\,dz$$

$$-\frac{1}{2}\int \frac{1}{z^2-4z+1}\,d(z^2-4z+1) = \int \frac{1}{u}\,du$$

$$-\frac{1}{2}\ln|z^2-4z+1| = \ln u + \ln c$$

$$\ln|z^2-4z+1|^{-\frac{1}{2}} = \ln uc$$

$$|z^2-4z+1|^{-\frac{1}{2}} = uc$$

$$\frac{1}{(z^2-4z+1)^{\frac{1}{2}}} = uc$$

$$z = \frac{v}{u} = \frac{y-2}{x+1}$$

$$\frac{1}{\left[\left(\frac{y-2}{x+1}\right)^2 - 4\frac{(y-2)}{(x+1)} + 1\right]^{\frac{1}{2}}} = (x+1)c$$

$$\frac{1}{\left[\frac{(y-2)^2}{(x+1)^2} - 4\frac{(y-2)}{(x+1)} + 1\right]} = (x+1)^2 c$$