เฉลย Midterm feed back

ข.1 $f(t) = t\cos 5t$

จากกฎคูณด้วย

$$\mathcal{L}\, t\, f(t) = -F'(s)$$

และกรณีทั่วไป

$$\mathcal{L}\, t^{k+1} f(t) = (-1)^{k+1} F^{(k+1)}(s)$$

ดิฟ

$$\mathcal{L} \cos 5t = \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$$

$$\mathcal{L}\, t\cos 5t = (-1)^1 \frac{d}{ds}\left(\frac{s}{s^2+25}\right)$$

$$= -\frac{\left[(s^2+25)\frac{ds}{ds} - s\frac{d(s^2+25)}{ds}\right]}{(s^2+25)^2}$$

$$= -\frac{\left[(s^2+25) - s(2s)\right]}{(s^2+25)^2} = \frac{-s^2-25+2s^2}{(s^2+25)^2}$$

$$= \frac{s^2-25}{(s^2+25)^2}$$ ##

1.2

$$\mathcal{L}\, f(t) = e^{-t} \cos 5t$$

on $\quad \mathcal{L}\, e^{at} f(t) = F(s-a)$

$$\mathcal{L}\, e^{-at} f(t) = F(s+a)$$

$$\mathcal{L} \cos 5t = \frac{s}{s^2+5^2} = \frac{s}{s^2+25}$$

$$\mathcal{L}\, e^{-t} \cos 5t = \frac{(s+1)}{(s+1)^2+25} \quad \checkmark$$

$$= \frac{(s+1)}{(s^2+2s+1)+25} = \frac{s+1}{s^2+2s+26}$$

2.1

$$\mathcal{L} F(s) = \mathcal{L} \frac{1}{(s+1)^2 (s+4)}$$

$$f(t) = \frac{A}{(s+1)^2} + \frac{B}{(s+1)^2} + \frac{C}{(s+4)}$$

$$A = \lim_{s\to -1} (s+1)^2 F(s) = \lim_{s\to -1} \cancel{(s+1)^2} \frac{1}{\cancel{(s+1)^2}(s+4)}$$

$$A = \lim_{s\to -1} \frac{1}{(s+4)} = \frac{1}{(-1+4)} = \frac{1}{3}$$

$$A_k = \lim_{s\to a} \left[\frac{1}{(m-k)!} \frac{d^{m-k}}{ds^{m-k}} (s-a)^m F(s) \right]$$

$m = 2$

$$A_2 = \lim_{s\to -1} \left[\frac{1}{(2-1)!} \frac{d^{2-1}}{ds^{2-1}} \cancel{(s+1)^2} \frac{1}{\cancel{(s+1)^2}(s+4)} \right]$$

$$B \; \cancel{A_2} = \lim_{s\to -1} 1 \frac{d}{ds} \frac{1}{(s+4)} = \lim_{s\to -1} \frac{d}{ds} (s+4)^{-1}$$

$$B = \lim_{s\to -1} (-1)(s+4)^{-2} \frac{d(s+4)}{ds}$$

$$B = \lim_{s\to -1} -\frac{1}{(s+4)^2} = -\frac{1}{(-1+4)^2} = -\frac{1}{3^2} = -\frac{1}{9}$$

$$C = \lim_{s \to -4} \cancel{(s+4)} \frac{1}{(s+1)^2\cancel{(s+4)}}$$

$$C = \lim_{s \to -4} \frac{1}{(s+1)^2}$$

$$C = \frac{1}{(-4+1)^2} = \frac{1}{9}$$

เทียบ สปส. $1 = A(s+4) + B(s+1)(s+4) + C(s+1)^2$

$$1 = As + 4A + B(s^2 + 4s + 1s + 4) + C(s^2 + 2s + 1)$$

$$1 = As + 4A + Bs^2 + 4Bs + Bs + 4B + Cs^2 + 2Cs + C$$

$$1 = As + 4A + Bs^2 + 5Bs + 4B + Cs^2 + 2Cs + C$$

$$1 = (B+C)s^2 + (A+5B+2C)s + 4A + 4B + C$$

$4A + 4B + C = 1$ —— (1)

$B + C = 0$

$B = -C$ —— (2)

$A + 5B + 2C = 0$ —— (3)

แทน (2) ใน (3)

$A + 5(-C) + 2C = 0$

$A - 5C + 2C = 0$

$A - 3C = 0$

$A - 3C = 0$

$A = 3C$ —— (4)

[illegible] (4) (2) [illegible] (1)

$$4(3C) + 4(-C) + C = 1$$

$$12C - 4C + C = 1$$

$$9C = 1$$

$$C = \frac{1}{9}$$

$$A = 3C = 3\,\frac{1}{9} = \frac{1}{3}$$

$$B = -C = -\frac{1}{9}$$

$$A = \frac{1}{3}\ , \quad B = -\frac{1}{9} \quad C = \frac{1}{9}$$

$$F(s) = \frac{1}{3}\,\frac{1}{(s+1)^2} - \frac{1}{9}\,\frac{1}{(s+1)} + \frac{1}{9}\,\frac{1}{(s+4)}$$

$$\mathcal{L}^{-1}F(s) = \frac{1}{3}\,\mathcal{L}^{-1}\frac{1}{(s+1)^2} - \frac{1}{9}\,\mathcal{L}^{-1}\frac{1}{(s+1)} + \frac{1}{9}\,\mathcal{L}^{-1}\frac{1}{(s+4)}$$

$$\boxed{\mathcal{L}^{-1}\frac{n!}{s^{n+1}} = t^n} \qquad \boxed{\mathcal{L}^{-1}\frac{1}{s-a} = e^{at}}$$

$$\mathcal{L}^{-1}\frac{1}{s^2} = \frac{1}{2!}\,t = \frac{1}{2}\,t \qquad \boxed{\mathcal{L}^{-1}\frac{1}{s+a} = e^{-at}}$$

$$\mathcal{L}^{-1}\frac{1}{(s+1)^2} = \frac{1}{2}\,t\,e^{-at}$$

$$\frac{1}{3}\mathcal{L}^{-1}\frac{1}{(s+1)^2} = \frac{1}{3}\left(\frac{1}{2}t e^{-t}\right) = \frac{1}{6}t e^{-t}$$ —— #

$$-\frac{1}{9}\mathcal{L}^{-1}\frac{1}{(s+1)} = -\frac{1}{9}e^{-t}$$

$$\frac{1}{9}\mathcal{L}^{-1}\frac{1}{(s+4)} = \frac{1}{9}e^{-4t}$$

$$\therefore f(t) = \mathcal{L}^{-1}\frac{1}{(s+1)^2(s+4)} = \frac{1}{6}t e^{-t} - \frac{1}{9}e^{-t} + \frac{1}{9}e^{-4t}$$ —— #

2-2

$$F(s) = \frac{5(s+10)}{s^2(s+3)(s+5)} = \frac{5s+50}{s^2(s+3)(s+5)}$$

$$\frac{5s+50}{s^2(s+3)(s+5)} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+3)} + \frac{D}{(s+5)}$$

$$A = \lim_{s\to 0} s^2\left[\frac{5(s+10)}{s^2(s+3)(s+5)}\right] = \frac{5(10)}{3\times 5} = \frac{50}{15} = \frac{10}{3}$$

$$A = \frac{10}{3}$$

$$B = \lim_{s\to 0}\frac{d}{ds}\left[\frac{5s+50}{(s+3)(s+5)}\right] =$$

$$\lim_{s\to 0}\frac{\left[(s+3)(s+5)\overset{\to 5}{\frac{d}{ds}(5s+50)} - (5s+\cancel{50})\frac{d}{ds}\cancel{}(s+3)(s+5)\right]}{\left[(s+3)(s+5)\right]^2}$$

$$s^2+5s+3s+15 = s^2+8s+15$$

$$\frac{d}{d}(s^2+8s+15) = 2s+8$$

$$\lim_{s\to 0}\left[\frac{5(s+3)(s+5) - (5s+50)(2s+8)}{\left[(s+3)(s+5)\right]^2}\right]$$

$$\lim_{s\to 0}\left[\frac{5(3)(5) - (50)(8)}{\left[(3)(5)\right]^2}\right] = \frac{-325}{225} = -\frac{65}{45} = -\frac{13}{9}$$

$$B = -\frac{13}{9}$$

$$C = \lim_{s\to -3}\cancel{(s+3)}\left[\frac{5(s+10)}{s^2\cancel{(s+3)}(s+5)}\right]$$

$$C = \lim_{s\to -3}\frac{5(s+10)}{s^2(s+5)} = \frac{5(-3+10)}{(-3)^2(-3+5)} = \frac{5\times 7}{9(2)} = \frac{35}{18}$$

$$= \frac{35}{18}$$

$$D = \lim_{s\to -5}\cancel{(s+5)}\left[\frac{5(s+10)}{s^2(s+3)\cancel{(s+5)}}\right] = \frac{5(-5+10)}{(-5)^2(-5+3)}$$

$$D = \frac{\cancel{(25)}}{\cancel{(25)}(-2)} = -\frac{1}{2}$$

$$\frac{5s+50}{s^2(s+3)(s+5)} = \frac{10}{3}\frac{1}{s^2} - \frac{13}{9}\frac{1}{s} + \frac{35}{18}\frac{1}{(s+3)} - \frac{1}{2}\frac{1}{(s+5)}$$

$$f(t) = \mathcal{L}^{-1}\left[\;\right] = \frac{10}{3}\mathcal{L}^{-1}\frac{1}{s^2} - \frac{13}{9}\mathcal{L}^{-1}\frac{1}{s} + \frac{35}{8}\mathcal{L}^{-1}\frac{1}{(s+3)} - \frac{1}{2}\mathcal{L}^{-1}\frac{1}{(s+5)}$$

$$f(t) = \frac{10}{3}\left[\frac{1}{2!}\mathcal{L}^{-1}\frac{2!}{s^{(1+1)}}\right] - \frac{13}{9}(1) + \frac{35}{8}e^{-3t} - \frac{1}{2}e^{-5t}$$

$$f(t) = \frac{10}{6}t - \frac{13}{9} + \frac{35}{8}e^{-3t} - \frac{1}{2}e^{-5t}$$

2-3

$$F(s) = \frac{2(s+1)}{s(s^2+s+2)} = \frac{A}{s} + \frac{Bs+C}{(s^2+s+2)}$$

$\cancel{s(s+1)(s+2) = s^2+2s-s-2}$

$\cancel{(s+1)(s-2) = s^2-2s+s-2}$

$$(2s+2) = A(s^2+s+2) + s(Bs+C)$$

$$2s+2 = As^2 + As + 2A + Bs^2 + Cs$$

$$2s+2 = (A+B)s^2 + (A+C)s + 2A$$

$A+B = 0$ ——— ①

$A+C = 2$ ——— ②

$2A = 2$ ——— ③

$A = \frac{2}{2} = 1$ ——— ④

แทน ④ ลงใน ①

$1+B = 0$

$B = -1$

แทน ④ ลงใน ②

$1+C = 2$

$C = 2-1 = 1$

5

$$F(s) = \frac{1}{s} + \frac{-s+1}{s^2+s+2}$$

$$= \frac{1}{s} - \frac{s}{s^2+s+2} + \frac{1}{s^2+s+2}$$

$$= \frac{1}{s} - \frac{s}{(s+\frac{1}{2})^2 + \frac{7}{4}} + \frac{1}{(s+\frac{1}{2})^2 + \frac{7}{4}}$$

$$= \frac{1}{s} - \left[\frac{(s+\frac{1}{2}) - \frac{1}{2}}{(s+\frac{1}{2})^2 + \frac{7}{4}} \right] + \frac{1}{(s+\frac{1}{2})^2 + \frac{7}{4}}$$

$$= \frac{1}{s} - \frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2 + \frac{7}{4}} + \frac{1}{2} \frac{1}{(s+\frac{1}{2})^2 + \frac{7}{4}} + \frac{1}{(s+\frac{1}{2})^2 + \frac{7}{4}}$$

$$= \frac{1}{s} - \frac{(s+\frac{1}{2})}{(s+\frac{1}{2})^2 + (\frac{\sqrt{7}}{2})^2} + \frac{1}{\cancel{2}} \frac{\cancel{2}}{\sqrt{7}} \frac{\frac{\sqrt{7}}{2}}{(s+\frac{1}{2})^2 + (\frac{\sqrt{7}}{2})^2} + \frac{2}{\sqrt{7}} \frac{\frac{\sqrt{7}}{2}}{(s+\frac{1}{2})^2 (\frac{\sqrt{7}}{2})^2}$$

$$= 1 - e^{-\frac{1}{2}t} \cos \frac{\sqrt{7}}{2} t + \frac{1}{\sqrt{7}} e^{-\frac{1}{2}t} \sin \frac{\sqrt{7}}{2} t + \frac{2}{\sqrt{7}} e^{-\frac{1}{2}t} \sin \frac{\sqrt{7}}{2} t$$

3.1

$$x''(t) + 5x'(t) + 4x(t) = e^{-t}u(t)$$

$$x'(0) = 0 \quad \text{και} \quad x(0) = 0$$

$$\mathcal{L}\left[x''(t) + 5x'(t) + 4x(t)\right] = \mathcal{L}\, e^{-t}u(t)$$

$$\left[s^2 X(s) - sX(0) - X'(0)\right] + 5\left[sX(s) - X(0)\right] + 4X(s) = \frac{1}{(s+1)}$$

$$s^2 X(s) - s\cancel{X(0)}^{0} - \cancel{X'(0)}^{0} + 5sX(s) - 5\cancel{X(0)}^{0} + 4X(s) = \frac{1}{(s+1)}$$

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$$s^2 X(s) + 5sX(s) + 4X(s) = \frac{1}{(s+1)}$$

$$X(s)(s^2 + 5s + 4) = \frac{1}{(s+1)}$$

$$X(s) = \frac{1}{(s+1)(s^2+5s+4)} = \frac{1}{(s+1)(s+4)(s+1)}$$

$$X(s) = \frac{A}{(s+1)} + \frac{B}{(s+4)} + \frac{C}{(s+1)}$$

$$\cancel{A = \lim_{s\to -1} (s+1)\frac{1}{(s+1)(s+4)(s+1)} = \frac{1}{(-1+4)(-1+1)}}$$

$$X(s) = \frac{1}{(s+1)^2(s+4)} = \frac{A}{(s+1)^2} + \frac{B}{(s+1)} + \frac{C}{(s+4)}$$

$$1 = A(s+4) + B(s+1)(s+4) + C(s+1)^2$$

$$1 = As + 4A + B(s^2 + 4s + s + 4) + C(s^2 + 2s + 1)$$

$$1 = As + 4A + Bs^2 + 4Bs + Bs + 4B + Cs^2 + 2Cs + C$$

$$1 = (B+C)s^2 + (A + \text{[illegible]}\ 5B + 2C)s + (4A + 4B + C)$$

$$1 = (B+C)s^2 + (A+5B+2C)s + (4A+4B+C)$$

$$B + C = 0 \quad \text{——} \quad (1)$$

$$(A + 5B + 2C) = 0 \quad \text{——} \quad (2)$$

$$4A + 4B + C = 1 \quad \text{——} \quad (3)$$

$$B = -C \quad \text{——} \quad (4)$$

[illegible] (4) [illegible] (2)

$$A + 5(-C) + 2C = 0$$

$$A - 5C + 2C = 0$$

$$A - 3C = 0 \quad \text{——} \quad (5)$$

[illegible] (4) [illegible] (3)

$$4A + 4(-C) + C = 1$$

$$4A - 4C + C = 1$$

$$4A - 3C = 1 \quad \text{——} \quad (6)$$

[illegible] (5) $\quad A = 3C \quad \text{——} \quad (7)$

[illegible] (7) [illegible] (6)

$$4(3C) - 3C = 1$$

$$12C - 3C = 1$$

$$9C = 1$$

$$C = \frac{1}{9}$$

$$B = -C = -\frac{1}{9} \longrightarrow A = 3C = 3\left(\frac{1}{9}\right.$$

$$A = \frac{1}{3}$$

$$x(s) = \frac{1}{3}\frac{1}{(s+1)^2} + \left(-\frac{1}{9}\right)\frac{1}{(s+1)} + \frac{1}{9}\frac{1}{(s+4)}$$

$$= \frac{1}{3}\frac{1}{(s+1)^2} - \frac{1}{9}\frac{1}{(s+1)} + \frac{1}{9}\frac{1}{(s+4)}$$

$$x(t) = \frac{1}{3}te^{-t} - \frac{1}{9}e^{-t} + \frac{1}{9}e^{-4t}$$

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(4)

$$f(t) = \begin{cases} 1 \to 0 < t < 2 \\ t-2 \to 2 < t < 3 \end{cases}$$

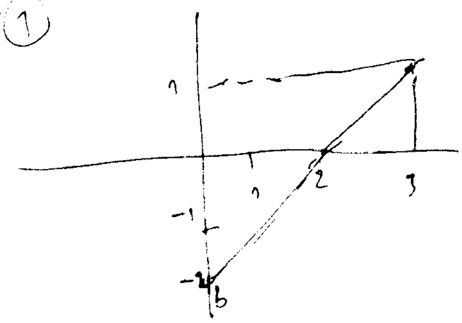
$f(t) = 5 - t \quad 4 < t < 5$

$f(t) = 1 \quad t > 5$

$f(t) = mt + \cancel{b}\ b$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{3-2} = \frac{1}{1} = 1$$

$f(t) = t + b$ — (1)



$y = mx + b$

$y = x + b$

$$\frac{1-b}{3-0} = 1$$

$1 - b = 3$

$-b = 3 - 1 = 2$

$b = -2$

$$\frac{1-b}{3-0} = \frac{1-0}{3-2} = 1$$

$1 - b = 1 \times 3 = 3$

$-b = 3 - 1 = 2$

$b = -2$ — (1)

$f(t) = t - 2$

$$\frac{0-b}{5-0} = -1$$

$$0 - b = -1 \times 5$$

$$-b = -5$$

$$b = +5$$

$$f(t) = -t + 5 = 5 - t$$

$$f(t) = u(t) - u(t-2) + (t-2)\left[u(t-2) - u(t-3)\right] + \left[u(t-3) - u(t-4)\right] + (5-t)\left[u(t-4) - u(t-5)\right] + u(t-5)$$

$$= u(t) - u(t-2) + (t-2)u(t-2) - (t-2)u(t-3) + (5-t)u(t-4) - (5-t)u(t-5) + u(t-5)$$

$$= u(t) - u(t-2) + tu(t-2) - 2u(t-2) - tu(t-3) + 2u(t-3)$$

$$5u(t-4) - tu(t-4) - 5u(t-5) + tu(t-5) + u(t-5)$$

$$\boxed{\begin{array}{c} \mathcal{L}\, u(t) = \frac{1}{s} \\ \mathcal{L}\left[f(t-b)u(t-b)\right] \\ = e^{-bs} F(s) \end{array}}$$

$$F(s) = \frac{1}{s} - \frac{1}{s} e^{-2s}$$

$$\mathcal{L}\, t u(t-2) \to -F'(s) = -\frac{d}{} e$$

$=$

$$f(t) = u(t) - u(t-2) + (t-2)\left[u(t-2) - u(t-3)\right] + \left[u(t-3) - u(t-\right.$$

$$+ (5-t)\left[u(t-4) - u(t-5)\right] + u(t-5)$$

$$= u(t) - u(t-2) + (t-2)\,u(t-2) - (t-2)\,u(t-3) + u(t-3)$$

$$- u(t-4) + (5-t)\,u(t-4) - (5-t)\,u(t-5) + u(t-5)$$

$$= u(t) - \underline{u(t-2)} + t\,u(t-2) \underline{- 2u(t-2)} - t\,u(t-3) \underline{+ 2u(t-3)}$$

$$\underline{+ u(t-3)} - \underline{u(t-4)} + \underline{5u(t-4)} - t\,u(t-4) \underline{- 5u(t-5)} + t\,u(t-5)$$

$$\underline{+ u(t-5)}$$

$$= u(t) - 3u(t-2) + t\,u(t-2) - t\,u(t-3) + 3u(t-3)$$

$$+ 4u(t-4) - t\,u(t-4) - 4u(t-5) + t\,u(t-5)$$

$$= u(t) - 3u(t-2) + (t-2+2)\,u(t-2) - (t-3+3)\,u(t-3) + 3u(t-3)$$

$$+ 4u(t-4) - (t-4+4)\,u(t-4) - 4u(t-5) + (t-5+5)\,u(t-5)$$

$$= u(t) - 3u(t-2) + (t-2)\,u(t-2) + 2u(t-2) - (t-3)\,u(t-3) - \cancel{3u(t-3)}$$

$$+ \cancel{3u(t-3)} + \cancel{4u(t-4)} - (t-4)\,u(t-4) - \cancel{4u(t-4)} - 4u(t-5)$$

$$+ (t-5)\,u(t-5) + 5u(t-5)$$

$$\mathcal{L}\,u(t) = \frac{1}{s}$$

$$\mathcal{L}\,(t-b)\,u(t-b) = e^{-bs}F(s)$$

$$\boxed{\mathcal{L}\,f(t-b)\,u(t-b) = e^{-b}F(s)}$$

$$\mathcal{L}\,t\,u(t-3) = \mathcal{L}\left[(t-3+3)\,u(t-3)\right]$$

$$= \mathcal{L}\left[(t-3)\,u(t-3) + 3u(t-3)\right]$$

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$$= e^{?}\mathcal{L}\,t + 3\,\mathcal{L}\,u(t-3)$$

$$= e^{-3s}\frac{1}{s^2} + 3e^{-3s}\frac{1}{s}$$

$$= u(t) - 3u(t-2) + (t-2)u(t-2) + 2u(t-2) - (t-3)u(t-3)$$

$$- (t-4)u(t-4) - 4u(t-5) + (t-5)u(t-5) + 5u(t-5)$$

$$F(s) = \frac{1}{s} - 3\frac{e^{-2}}{s} + \frac{e^{-2}}{s^2} + 2\frac{e^{-2}}{s} - \frac{e^{-3s}}{s^2} - \frac{e^{-4s}}{s^2} - 4e^{-5s} + \frac{e^{-5s}}{s^2}$$

$$5\frac{e^{-5s}}{s}$$

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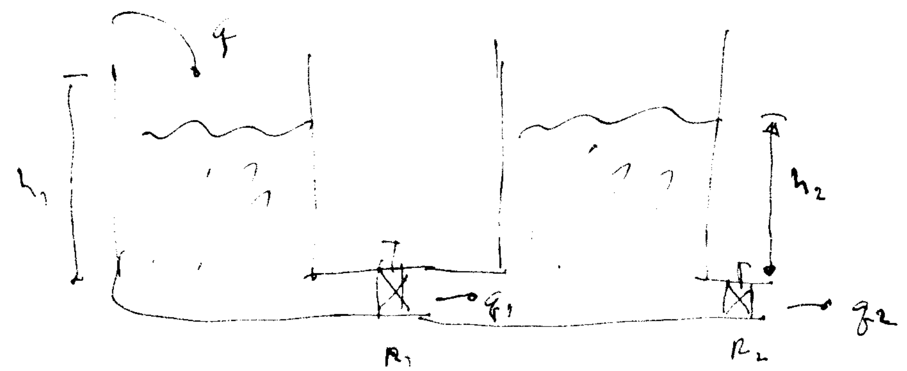
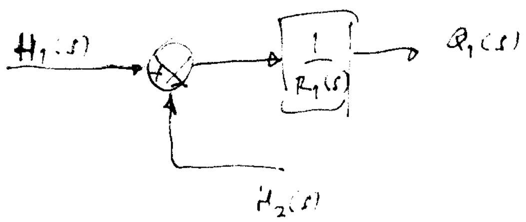
(5)



$$\frac{h_1 - h_2}{R_1} = q_1$$

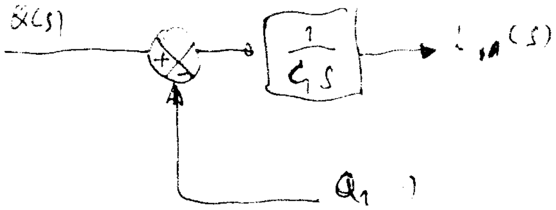
$$\frac{H_1(s) - H_2(s)}{R_1(s)} = Q_1(s) \qquad \text{——} \quad (1)$$



$$C_1 \frac{dh_1}{dt} = q - q_1$$

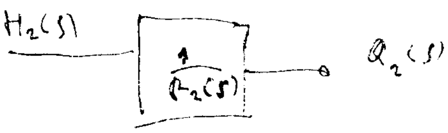
$$C_1 s H_1(s) = Q(s) - Q_1(s) \quad \text{——— (2)}$$

$$H_1(s) = \frac{Q(s) - Q_1(s)}{C_1 s} \quad \text{——— (3)}$$



$$q_2 = \frac{h_2}{R_2}$$

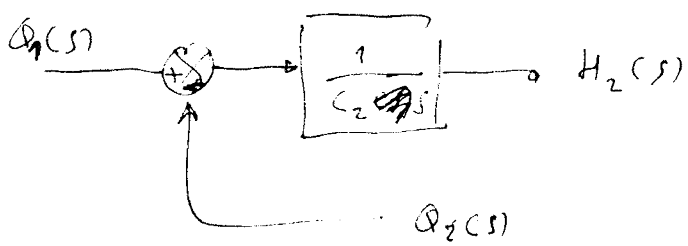
$$Q_2(s) = \frac{H_2(s)}{R_2(s)}$$



$$C_2 \frac{dh_2}{dt} = q_1 - q_2$$

$$C_2 s H_2(s) = Q_1(s) - Q_2(s)$$

$$H_2(s) = \frac{Q_1(s) - Q_2(s)}{C_2 s} \quad \text{——— (4)}$$



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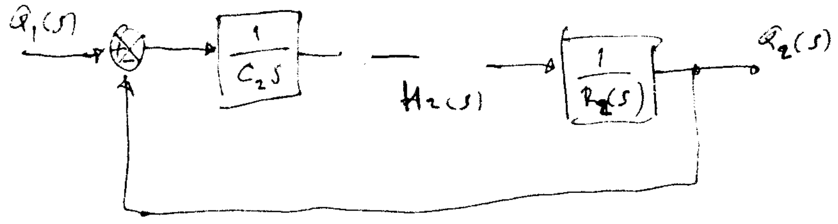
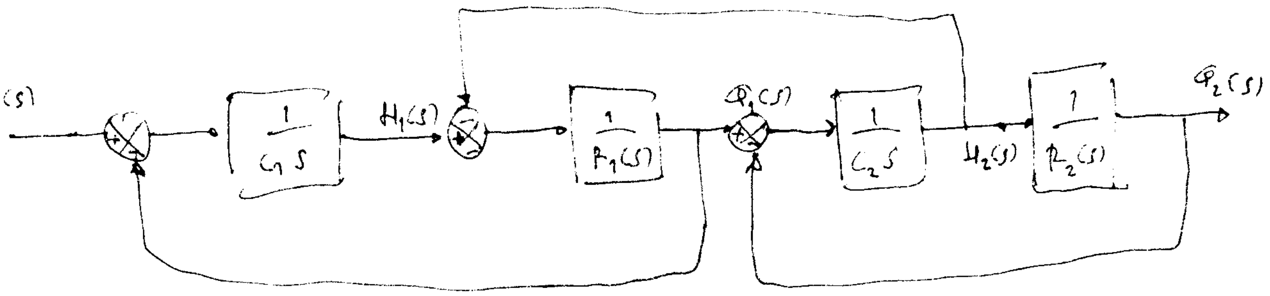
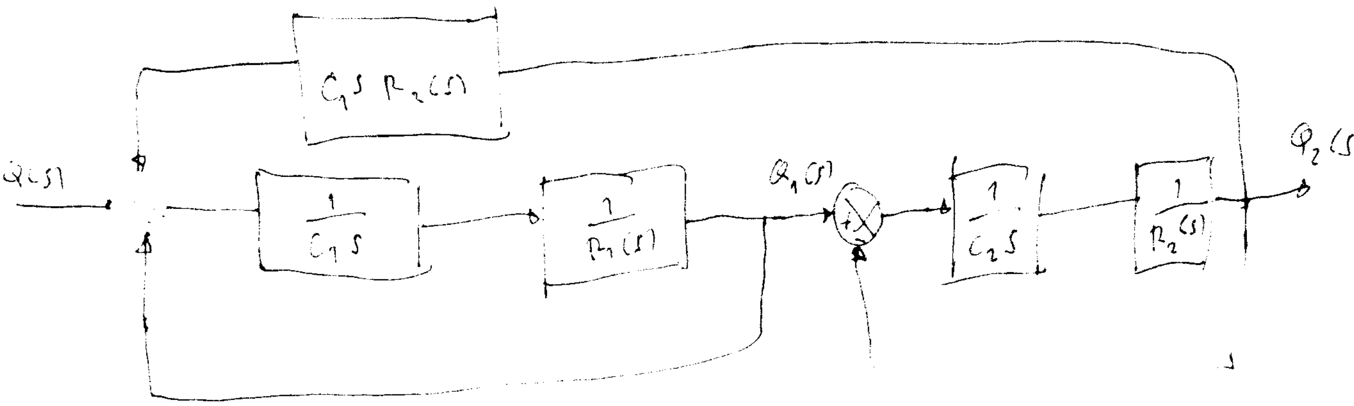
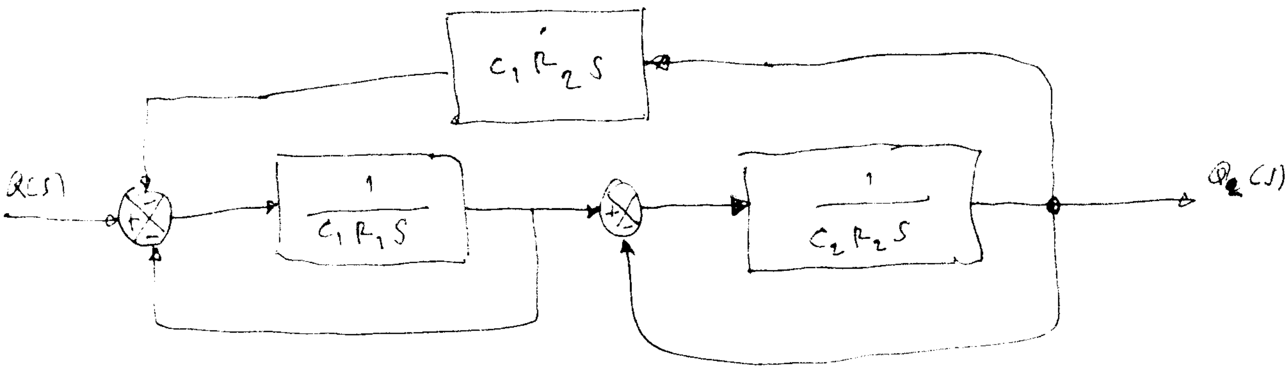
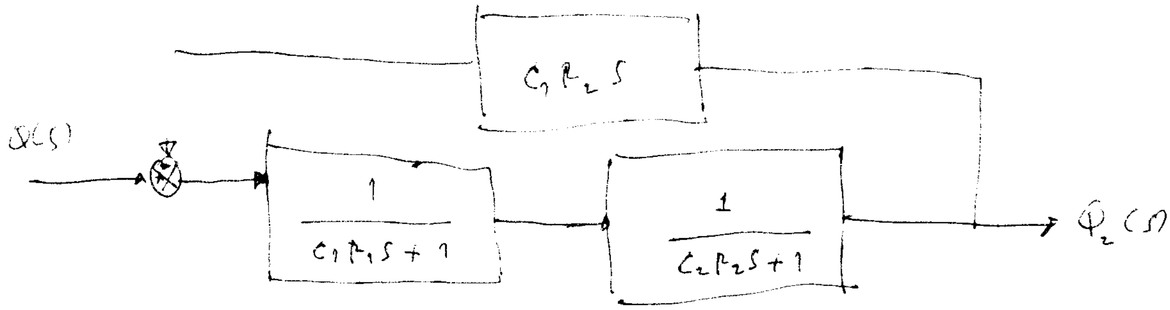


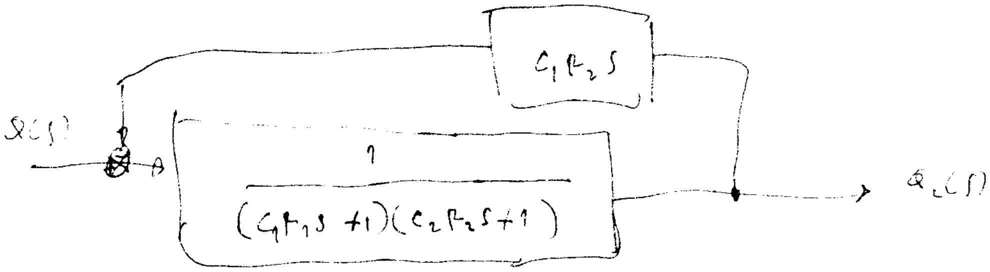


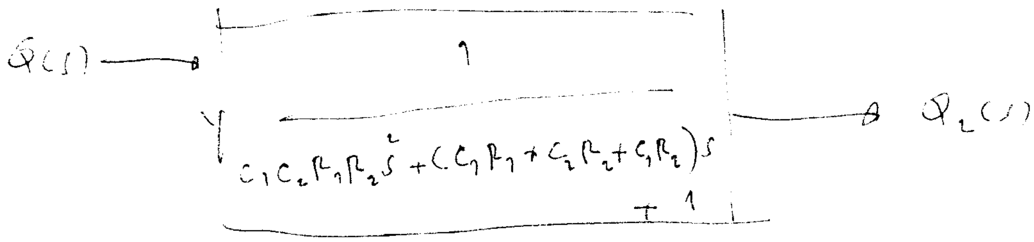




$$\frac{\frac{1}{C_1 R_1 s}}{1 + \frac{1}{C_1 R_1 s}} = \frac{\left[\frac{1}{\cancel{C_1 R_1 s}}\right]}{\left[\frac{C_1 R_1 s + 1}{\cancel{C_1 R_1 s}}\right]} = \frac{1}{C_1 R_1 s + 1}$$





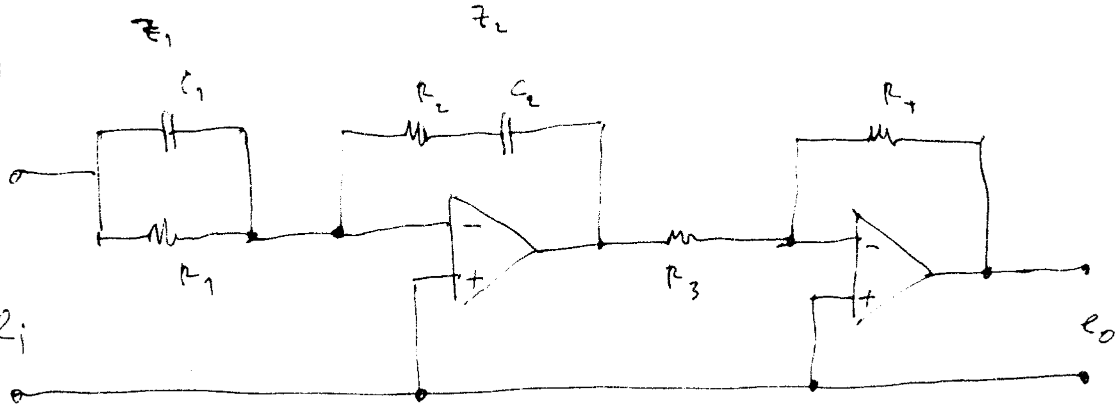


$$\frac{\left[\frac{1}{(C_1R_1s+1)(C_2R_2s+1)}\right]}{1+\left[\frac{1}{(C_1R_1s+1)(C_2R_2s+1)}\right](C_1R_2s)}$$

$$=\frac{\left[\frac{1}{(C_1R_1s+1)\cancel{(C_2R_2s+1)}}\right]}{\left[\frac{(C_1R_1s+1)(C_2R_2s+1)+C_1R_2s}{(C_1R_1s+1)\cancel{(C_2R_2s+1)}}\right]}=\frac{1}{C_1C_2R_1R_2s^2+C_1R_1s+C_2R_2s+C_1R_2s+1}$$

$$=\frac{1}{C_1C_2R_1R_2s^2+(C_1R_1+C_2R_2+C_1R_2)s+1}$$

(b)



$$\frac{1}{j\omega C} = \frac{1}{sC} \qquad j\omega L = sL$$

$$Z_1 = \frac{R_1 \times \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{\left(\frac{R_1}{\cancel{sC_1}}\right)}{\left(\frac{C_1R_1s+1}{\cancel{sC_1}}\right)} = \frac{R_1}{(C_1R_1s+1)}$$

$$Z_2 = R_2 + \cancel{sC_2}\,\frac{1}{sC_2} = \frac{R_2C_2s+1}{sC_2}$$

$$\text{Gain}_1 = -\cancel{B}\,\frac{Z_2}{Z_1} = -\frac{\left[\frac{(R_2C_2s+1)}{sC_2}\right]}{\left[\frac{R_1}{C_1R_1s+1)}\right]}$$

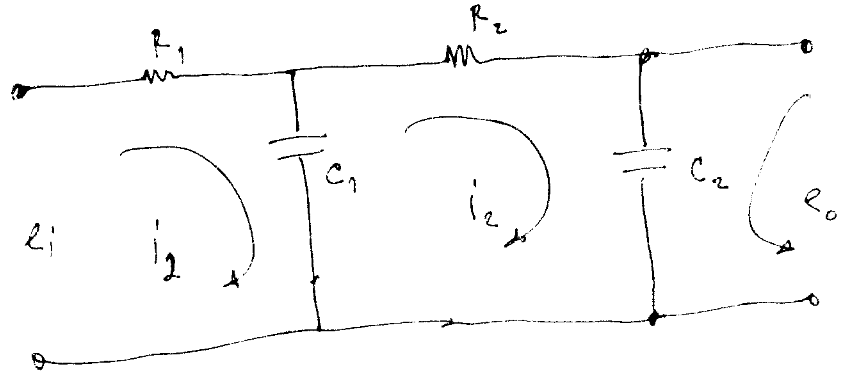
$$\text{Gain}_1 = -\frac{(R_2C_2s+1)(C_1R_1s+1)}{R_1C_2s}$$

$$\text{Gain}_2 = -\frac{R_4}{R_3}$$

$$e_o = \text{Gain}_1\, \text{Gain}_2\, e_i$$

$$= \left[-\frac{R_4}{R_3}\right]\left[-\frac{(R_2C_2s+1)(C_1R_1s+1)}{R_1C_2s}\right]e_i$$

$$\frac{E_o(s)}{E_i(s)} = \frac{R_4}{R_3}\frac{R_2}{R_1}\left[\frac{(R_2C_2s+1)(C_1R_1s+1)}{R_1C_2s}\right] \text{ — b}$$



$$e_i = R_1 i_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

$$E_i(s) = R_1 I_1(s) + \frac{I_1(s) - I_2(s)}{C_1 s} \quad \text{— (1)}$$

$$i_2 R_2 + \frac{1}{C_1}\int (i_2 - i_1)\,dt + \frac{1}{C_2}\int i_2\,dt = 0$$

$$I_2(s)R_2 + \frac{I_2(s) - I_1(s)}{sC_1} + \frac{I_2(s)}{sC_2} = 0 \quad \text{— (2)}$$

$$e_o = \frac{1}{C_2}\int i_2\,dt$$

$$E_o(s) = \frac{1}{C_2 s} I_2(s) \quad \text{———} \quad ③$$

จาก ②

$$I_2(s)R_2 + \frac{I_2(s)}{sC_1} - \frac{I_1(s)}{sC_1} + \frac{I_2(s)}{sC_2} = 0$$

$$I_2(s)\left(R_2 + \frac{1}{sC_1} + \frac{1}{sC_2}\right) = \frac{I_1(s)}{sC_1}$$

$$I_2(s)\left[\frac{R_2C_1C_2s + C_2 + C_1}{C_1C_2s}\right] = \frac{I_1(s)}{sC_1}$$

$$I_1 = (\cancel{sC_1})\left[\frac{R_2C_1C_2s + C_2 + C_1}{\cancel{C_1}C_2\cancel{s}}\right] I_2(s)$$

$$I_1(s) = \left[\frac{R_2C_1C_2s + C_2 + C_1}{C_2}\right] I_2(s) \quad \text{———} \quad ④$$

จาก ①

$$E_i(s) = R_1 I_1(s) + \frac{I_1(s)}{C_1 s} - \frac{I_2(s)}{C_1 s}$$

$$E_i(s) = I_1(s)\left[R_1 + \frac{1}{C_1 s}\right] - \frac{I_2(s)}{C_1 s}$$

$$E_i(s) = I_1(s)\left[\frac{R_1C_1s + 1}{C_1 s}\right] - \frac{I_2(s)}{C_1 s} \quad \text{———} \quad ⑤$$

แทน ④ และ ⑤

$$E_i(s) = \left[\frac{R_2C_1C_2s + C_2 + C_1}{C_2}\right]\left[\frac{R_1C_1s + 1}{C_1 s}\right] I_2(s) - \frac{I_2(s)}{C_1 S}$$

$$E_i(s) = \left[\left(\frac{(R_2C_1C_2s + C_2 + C_1)}{C_2}\right)\left(\frac{R_1C_1s+1}{C_1s}\right) - \frac{1}{C_1s}\right] I_2(s)$$

$$= \frac{(R_2C_1C_2s + C_2 + C_1)(R_1C_1s+1) - C_2}{C_1C_2s} I_2(s)$$

$$= \frac{(R_2C_1C_2s)(R_1C_1s+1) + C_2(R_1C_1s+1) + C_1(R_1C_1s+1) - C_2}{C_1C_2s} I_2(s)$$

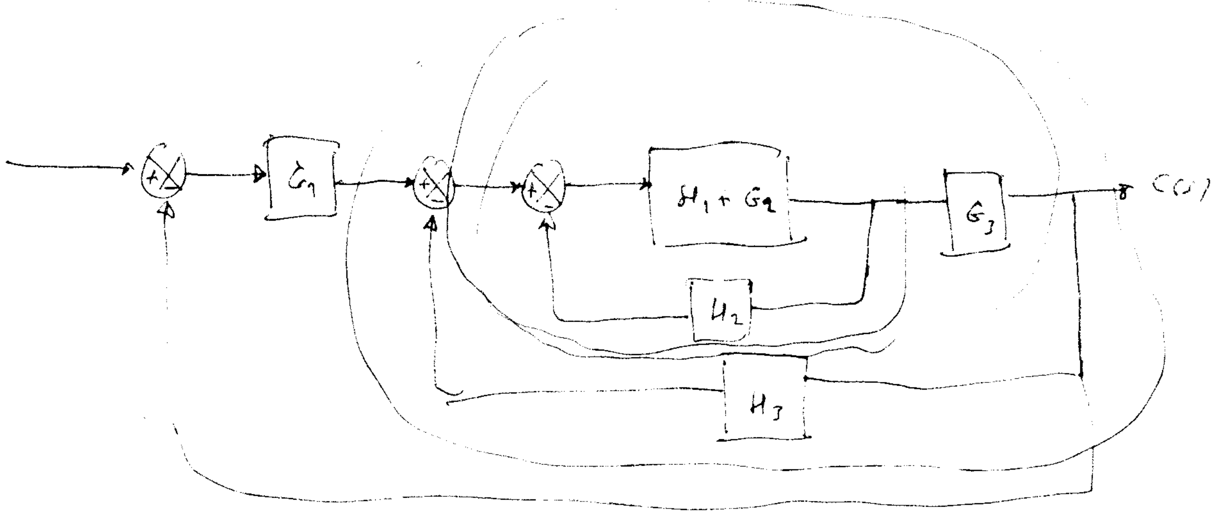
$$= \left[\frac{R_2\cancel{C_1^2}C_2s^2 + R_2\cancel{C_1}C_2s + C_2R_1\cancel{C_1}s + \cancel{C_2} + \cancel{C_1}R_1C_1s + \cancel{C_1} - \cancel{C_2}}{\cancel{C_1}C_2s}\right] I_2(s)$$

$$= \left[\frac{R_2C_1C_2s^2 + R_2C_2s + C_2R_1s + R_1C_1s + 1}{C_2s}\right] I_2(s)$$

$$E_i(s) = \left(\frac{R_2C_1C_2s^2 + (R_2C_2 + C_2R_1 + R_1C_1)s + 1}{C_2s}\right) I_2(s)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{⑤}{⑥⑦} = \frac{1}{\left[(R_2C_1C_2s^2 + (R_2C_2 + C_2R_1 + R_1C_1)s + 1\right]}$$

(7)



Step 1

$$\frac{(H_1+G_2)}{1+H_2(H_1+G_2)} = \frac{(H_1+G_2)}{1+H_1H_2+G_2H_2}$$

Step 2

$$\frac{(H_1+G_2)\,G_3}{1+H_1H_2+G_2H_2} = \frac{(H_1G_3+G_2G_3)}{1+H_1H_2+G_2H_2}$$

Step 3

$$\frac{\dfrac{(H_1G_3+G_2G_3)}{(1+H_1H_2+G_2H_2)}}{1+H_3\left[\dfrac{H_1G_3+G_2G_3}{(1+H_1H_2+G_2H_2)}\right]}$$

$$= \frac{\left[\dfrac{H_1G_3+G_2G_3}{\cancel{(1+H_1H_2+G_2H_2)}}\right]}{\left[\dfrac{(1+H_1H_2+G_2H_2)+H_1H_3G_3+G_2G_3H_3}{\cancel{(1+H_1H_2+G_2H_2)}}\right]}$$

$$= \frac{H_1G_3+G_2G_3}{G_2G_3H_3+H_1H_3G_3+G_2H_2+H_1H_2+1}$$

Step 4

$$\text{Gain} = \frac{G_1G_3H_1 + G_2G_3G_1}{G_2G_3H_3 + H_1H_3G_3 + G_2H_2 + H_1H_2 + 1}$$

Step 8

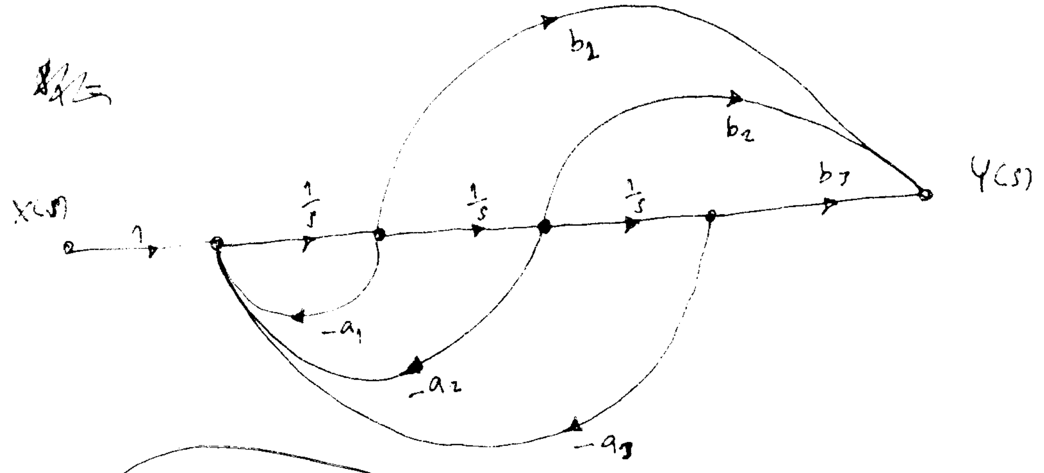
$$= \frac{\left[\dfrac{G_1G_3H_1 + G_2G_3G_1}{G_2G_3H_3 + H_1H_3G_3 + G_2H_2 + H_1H_2 + 1}\right]}{1 + \left[\dfrac{G_1G_3H_1 + G_1G_2G_3}{(G_2G_3H_3 + H_1H_3G_3 + G_2H_2 + H_1H_2 + 1)}\right]}$$

$$= \cancel{G_1G_3H_1 + G_2G_2 + 1}$$

$$= \frac{G_1G_2G_3 + G_1G_3H_1}{G_2G_3H_3 + H_1H_3G_3 + G_2H_2 + H_1H_2 + G_1G_3H_1 + G_1G_2G_3 + 1}$$

(8) $$\frac{Y(s)}{X(s)} = \frac{1}{\Delta}\left(P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + \dots\right)$$

Sol =



$P_3 = \dfrac{b_3}{s^3}$

$P_1 = \dfrac{b_1}{s}$

$P_2 = \dfrac{b_2}{s^2}$

$P_1 = \dfrac{b_1}{s}$

$P_2 = \dfrac{b_2}{s^2}$

$P_3 = \dfrac{b_3}{s^3}$

$L_1 = -\dfrac{a_1}{s}$

$L_2 = -\dfrac{a_2}{s^2}$

$L_3 = -\dfrac{a_3}{s^3}$

$$\Delta = 1 - (L_1 + L_2 + L_3) + 0$$

$$\Delta = 1 - (L_1 + L_2 + L_3) = 1 - \left(-\frac{a_1}{s} - \frac{a_2}{s^2} - \frac{a_3}{s^3}\right)$$

$$\Delta = 1 + \left(\frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{s^3}\right)$$

ทุก Loop แตะกับ P_1

$$\Delta_1 = 1$$

ทุก loop แตะกับ P_2

$$\Delta_2 = 1$$

ทุก loop แตะกับ P_3

$$\Delta_3 = 1$$

$$\frac{Y(s)}{X(s)} = \frac{1}{\Delta}\left(P_1 + P_2 + P_3\right)$$

$$= \frac{\frac{b_1}{s} + \frac{b_2}{s^2} + \frac{b_3}{s^3}}{1 + \left(\frac{a_1}{s} + \frac{a_2}{s^2} + \frac{a_3}{s^3}\right)} = \frac{\frac{b_1 s^2 + b_2 s + b_3}{s^3}}{1 + \frac{a_1 s^2 + a_2 s + a_3}{s^3}}$$

$$= \frac{\left[\frac{b_1 s^2 + b_2 s + b_3}{\cancel{s^3}}\right]}{\left[\frac{s^3 + a_1 s^2 + a_2 s + a_3}{\cancel{s^3}}\right]}$$

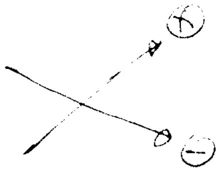
$$\frac{Y(s)}{X(s)} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}$$ —— ตอบ

(3)

$$\frac{C(s)}{R(s)} = \frac{15s^2 - 25s - 20}{(s^2-1)(s^2+8s+2s+16)}$$

$$= \frac{15s^2 - 25s + 20}{s^4 + 8s^3 + 2s^3 + 16s^2 - s^2 - 8s - 2s - 16}$$

$$= \frac{15s^2 - 25s - 20}{s^4 + 10s^3 + 15s^2 - 10s - 16}$$



s^4	1	15	−16
s^3	10	−10	0
s^2	b_1	b_2	b_3
s^1	c_1	c_2	c_3
s^0	D_1	D_2	D_3

$$b_1 = \frac{(10)(15) + (10)(1)}{10}$$

$$b_1 = \frac{150 + 10}{10} = \frac{160}{10} =$$

$$b_2 = \frac{-160 - 0}{10} = -1$$

$$b_3 = \frac{0 - 0}{10} = 0$$

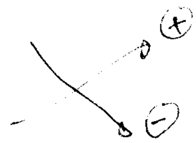
$$c_1 = \frac{(10)(-10) - (10)(-16)}{16} = \frac{-160 + 160}{16} = 0$$

$$c_2 = \frac{0 - 0}{16} = 0 \quad , c_3 = 0$$

$c_1 = 0$ $\quad c_2 = 0$ $\quad c_3 = 0$

จากแถว s^2

$$\longrightarrow A(s) = 16s^2 - 16s = 0$$



$$\frac{dA(s)}{ds} = 32s - 16$$

s^4	1	15	-16
s^3	10	-10	0
s^2	16	-16	0
s^1	32	-16	0
s^0	a_1	a_2	a_3

$$a_1 = \frac{(32)(-16) - (16)(-16)}{32} = -8$$

$$a_2 = \frac{0 - 0}{32} = 0$$

มีการเปลี่ยน ระบบไม่เสถียร.