

1) เหยือกน้ำ 2 ลูก 1 ลูก มีหมายเลขบนลูกน้ำคือ A มีหมายเลขที่ลูกน้ำที่ขึ้นหน้าคือเลขเป็นคี่
 B มีหมายเลขที่ลูกน้ำที่ขึ้นหน้าคือเลขเป็นคู่ 1 ลูก ขุดน้ำจากเหยือกน้ำ

ก) $P(A)$ ข) $P(B)$ ค) $P(A \cap B)$

ง) $P(A \cup B)$ จ) $P(B|A)$ ฉ) A หรือ B มีหมายเลขลูกน้ำเป็นคี่

วิธีทำ $S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$

ก) $P(A)$

$A = \{ (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \}$

$P(A) = \frac{6}{36} = \frac{1}{6} *$

ข) $P(B)$

$B = \{ (1,4), (2,4), (3,4), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,4), (6,4) \}$

$P(B) = \frac{11}{36} *$

ค) $P(A \cap B) = \{ (3,4), (4,3) \}$

$= \frac{2}{36} = \frac{1}{18} *$

ง) $P(A \cup B) = \{ (1,4), (1,6), (2,4), (2,5), (3,4), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,2), (5,4), (6,1), (6,4) \}$

$= \frac{15}{36} *$

a) $P(B|A)$

$$= \frac{P(A \cap B)}{P(A)}$$

$$= \frac{\left(\frac{1}{36}\right)}{\left(\frac{1}{6}\right)}$$

$$= \frac{1}{3} \quad \checkmark$$

b) A and B independent \checkmark

3) There are 100 users. A has 30 users, B has 30 users, C has 50 users. A has 5% errors, B has 4% errors, C has 2% errors. E = error.

25%

A has 30 users 5%

B has 30 users 4%

C has 50 users 2%

E = error

$$P(B|E) = \frac{P(B)P(E|B)}{P(A)P(E|A) + P(B)P(E|B) + P(C)P(E|C)}$$

$$= \frac{(0.3)(0.04)}{(0.3)(0.05) + (0.3)(0.04) + (0.5)(0.02)}$$

$$= \frac{0.012}{0.01 + 0.012 + 0.01}$$

$$= \frac{0.012}{0.032}$$

$$= 0.375$$

$$= \frac{3}{8} \quad \checkmark$$

$$= \frac{0.012}{0.032} = 0.375$$

$$0.375$$

$$P(A) = \frac{30}{100} = 0.3, P(E|A) = \frac{5}{100} = 0.05$$

$$P(B) = \frac{30}{100} = 0.3, P(E|B) = \frac{4}{100} = 0.04$$

$$P(C) = \frac{50}{100} = 0.5, P(E|C) = \frac{2}{100} = 0.02$$

3) ถ้าความน่าจะเป็นที่ชายคนหนึ่งจะสูบบุหรี่คือ 0.80 หรือ 80% หรือ 5 คน

ก) จงเขียนฟังก์ชันความน่าจะเป็นของจำนวนคนที่สูบบุหรี่

ข) จงหาค่าคาดหวัง และ ความแปรปรวนของจำนวนคนที่สูบบุหรี่

วิธีทำ. $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$n = 5, p = 0.80$

แทนค่า

$f(x) = \binom{5}{x} (0.80)^x (1-0.80)^{5-x}$; $x = 0, 1, 2, 3, 4, 5$ *

x	f(x)
0	$\binom{5}{0} (0.80)^0 (1-0.80)^{5-0} = (1)(1)\left(\frac{1}{3125}\right) = \frac{1}{3125}$
1	$\binom{5}{1} (0.80)^1 (1-0.80)^{5-1} = (5)(0.80)\left(\frac{1}{625}\right) = \frac{4}{625}$
2	$\binom{5}{2} (0.80)^2 (1-0.80)^{5-2} = (10)\left(\frac{16}{25}\right)\left(\frac{1}{125}\right) = \frac{32}{625}$
3	$\binom{5}{3} (0.80)^3 (1-0.80)^{5-3} = (10)\left(\frac{64}{125}\right)\left(\frac{1}{25}\right) = \frac{128}{625}$
4	$\binom{5}{4} (0.80)^4 (1-0.80)^{5-4} = (5)\left(\frac{256}{625}\right)(0.20) = \frac{256}{625}$
5	$\binom{5}{5} (0.80)^5 (1-0.80)^{5-5} = (1)\left(\frac{1024}{3125}\right)(1) = \frac{1024}{3125}$
	รวม = 1 *

ค) ค่าคาดหวัง = $E(x)$

$E(x) = np$

$n = 5, p = 0.80$

$E(x) = 5(0.80)$

$= 4$ *

ค่าความแปรปรวน = $V(x)$

$V(x) = np(1-p)$

$= [5(0.80)](1-0.80)$

$= 4(0.20) = 0.8$ *

(A) จำนวนสินค้าทั้งหมด 1000 ชิ้น จำนวนสินค้าที่ชำรุด 300 ชิ้น
 จำนวนสินค้าที่เลือก 10 ชิ้น จำนวนสินค้าที่ชำรุดที่เลือก 3 ชิ้น

สูตร:
$$P(X=x) = \frac{\binom{r}{x} \binom{N-r}{n-x}}{\binom{N}{n}}$$

$N = \text{จำนวนสินค้าทั้งหมด} = 1000 \text{ ชิ้น}$

$r = \text{จำนวนสินค้าที่ชำรุด} = 300 \text{ ชิ้น}$

$n = 10$

แทนค่า:
$$P(X=3) = \frac{\binom{300}{3} \binom{1000-300}{10-3}}{\binom{1000}{10}}$$

$$= \frac{\binom{300}{3} \binom{700}{7}}{\binom{1000}{10}}$$

$= 0.26$

✘



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$a = \text{rate of loss (in } \text{m}^3 \text{ per second)}$

$$\lambda = \frac{90}{60} = 1.5 \text{ s}^{-1}$$

for poisson process $\gamma = 4$

- $\Gamma(1) = 1$
- $\Gamma(2) = 1$
- $\Gamma(n+1) = n \Gamma(n)$
- $\Gamma(n) = (n-1) \Gamma(n-1)$
- $\Gamma(n) = (n-1)!$
- $\Gamma(1) = 1!$
- $\Gamma(2) = 2!$
- $\Gamma(3) = 3!$
- $\Gamma(n+1) = n!$
- $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$$f(x) = \frac{\lambda (\lambda x)^{\gamma-1} e^{-\lambda x}}{\Gamma(\gamma)} = \frac{\lambda (\lambda x)^{\gamma-1} e^{-\lambda x}}{(\gamma-1)!}$$

$$f(t) = \frac{\lambda (\lambda t)^{\gamma-1} e^{-\lambda t}}{\Gamma(\gamma)}$$

~~$\Gamma(\gamma) = (\gamma-1)!$~~

$$f(t) = \frac{1.5 (1.5t)^{4-1} e^{-1.5t}}{(4-1)!}$$

$$f(t) = \frac{1.5 (1.5t)^3 e^{-1.5t}}{3!}$$

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$$f(t) = \frac{1.5 (1.5)^3 t^3 e^{-1.5t}}{3!}$$

$$f(t) = 0.84375 t^3 e^{-1.5t}$$

or as $f(t) = \frac{1}{4} e^{-\frac{3}{2}t} \rightarrow t > 0$

8

$$E(x) = \frac{\gamma}{\lambda}$$

$$E(t) = \frac{4}{1.5} = 2.666 = \frac{8}{3} = 2.666$$

$$P(T < t) = 1 - \sum_{k=0}^{r-1} \frac{e^{-\lambda t} (\lambda t)^k}{k!} = \dots$$

a. $P(T < 2) \rightarrow r = 4$

$$\lambda t = 1.5 \times 2 = 3$$

$$P(T < 2) = 1 - \sum_{k=0}^3 \frac{e^{-3} (3)^k}{k!}$$

$$= 1 - e^{-3} \left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!} + \frac{3^3}{3!} \right)$$

$$= 1 - \frac{1}{e} (1 + 3 + 4.5 + 4.5)$$

$$= 1 - 0.04978 (12)$$

$$= 1 - 0.6492$$

$$P(T < 2) = 0.3529 \quad \text{————— } \checkmark$$

0.3529.

b. $\lambda t = 0.3528$ ————— $\frac{10^6 \times 0.3528}{10^6} = 352.8$

đây là (đơn vị) của...

10
2

3

$$\hat{a} = \frac{90}{60} = 1.5 \text{ s}^{-1} \text{ m}^{-1}$$

$\gamma = 1 \rightarrow$ exponential!

$$f(t) = a e^{-at} \rightarrow t) 0$$

11. $f(t) = 1.5 e^{-1.5t} \rightarrow t) 0$ ✓

12) sum $P(T \leq 3) \rightarrow \gamma = 1$

$$P(T \leq t) = 1 - \sum_{k=0}^{\gamma-1} \frac{e^{-at} (at)^k}{k!}$$

$$P(T \leq t) = 1 - a e^{-at}$$

$$1 \leq 1 - 1.5 e^{-1.5(3)}$$

$$1 = 1 - 1.5 (e^{-4.5})$$

$$1 = 1 - 0.0166$$

$$P(T \leq 3) \leq 0.9833$$

13)

$$E(T) = \frac{1}{\frac{1}{2}} = \frac{1}{(1.5)^2} = 0.444$$

0.666 ✓

2.

$$P(T \leq 3) = 1 - (1.5 e^{-1.5(3)})$$

$$= 1 - 1.5 \left(\frac{1}{90.019} \right)$$

$$= 1 - 1.5 (0.01108)$$

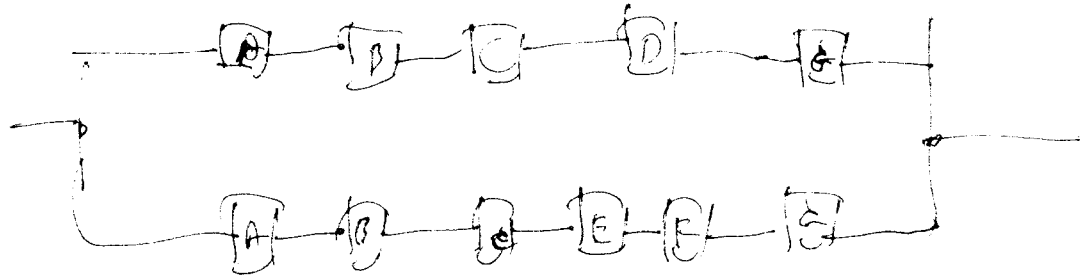
$$= 1 - 0.01666$$

$$= 0.983$$

Q1) or $E(T) = \frac{1}{\lambda} = \frac{1}{1.5} = 0.6666 = \frac{2}{3}$ — 1/2

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die-set ungenügend → über
57h 10 s. w. u. 470 14.



$$A = 0.9, B = 0.85, C = 0.8, D = 0.75, E = 0.7, F = 0.65, \delta = 0.6$$

~~P(A)P(B)~~

$$Wahrsch. d. d. = 1 - (1 - P_A P_B P_C P_D P_E) (1 - P_A P_B P_C P_E P_F P_G)$$

$$= 1 - (1 - 0.9 \times 0.85 \times 0.8 \times 0.75 \times 0.6) (1 - 0.9 \times 0.85 \times 0.8 \times 0.65 \times 0.7 \times 0.6)$$

$$= 1 - (1 - 0.2754) (1 - 0.167076)$$

$$= 1 - (0.7246) (0.8329)$$

$$= 1 - 0.6038$$

$$= 0.39646$$

Summe

0.808

~~0.808~~

8) $\sqrt{5}$ 24.2 → 2. 8. 8. 8. 8. 8. 8. 8. 8. 8. 8. 8.

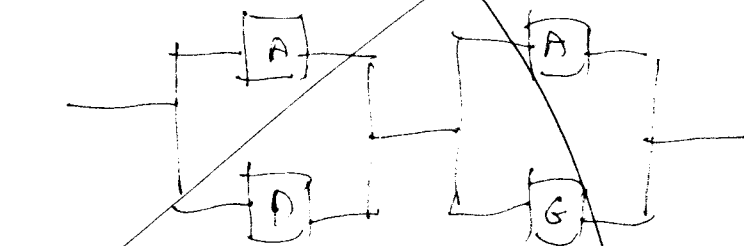
A	✓	AB	✓
B	✓	AC	✓
C	✓	AD	✓ → *
D	✓	AE	✓
E	✓	AF	✓
F	✓	AG	✗ → a
G	✓		

- AB C ✓
- AB D ✗ → *
- AB E ✓
- AB F ✓
- AB G ✗ → *

- ABC D → *
- ABC E ✓
- ABC F ✓
- ABC G ✗

- ABCDE — ✗
- BCDEF — ✗
- ABC DG — ✗

- ~~ABCDEF ✓ →~~
- ~~BCDEG ✗ →~~
- ~~ABCDEF G ✗ →~~



$$\begin{aligned} \text{muvij } 180 \text{ } \dot{n}_0 &= [1 - (1 - p_A)(1 - p_D)] [1 - (1 - p_A)(1 - p_G)] \\ &= [1 - (1 - 0.5)(1 - 0.75)] [1 - (1 - 0.5)(1 - 0.6)] \end{aligned}$$

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$$= (1 - (0.1 \times 0.25)) (1 - (0.1)(0.4))$$

$$= (1 - 0.025) (1 - 0.04)$$

$$= 0.975 \times 0.96$$

$$= 0.936$$

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$A = 0.9$

$B = 0.85$

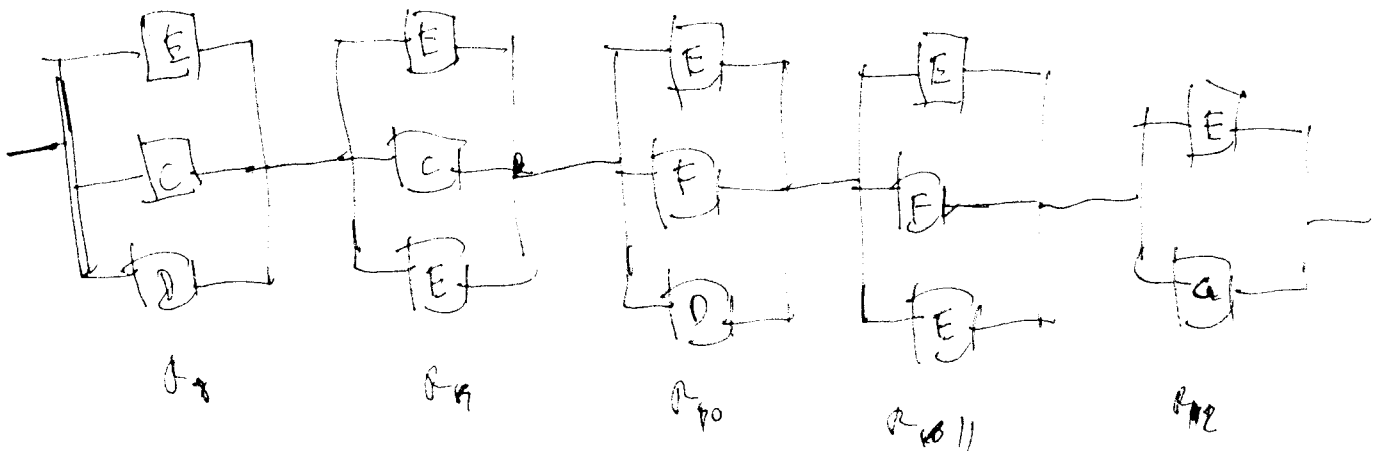
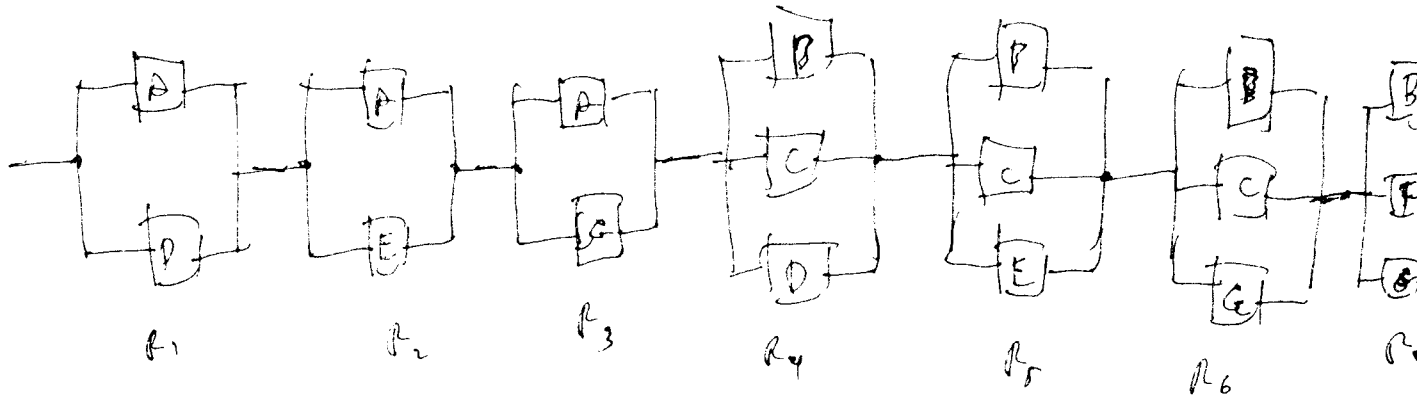
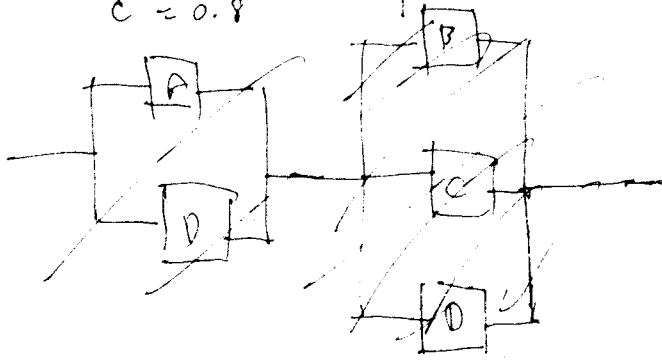
$C = 0.8$

$D = 0.75$

$E = 0.7$

$F = 0.65$

$G = 0.6$



$$R_1 = [1 - (1 - P_A)(1 - P_D)] = [1 - (1 - 0.9)(1 - 0.75)] = 0.975$$

$$R_2 = [1 - (1 - P_A)(1 - P_E)] = [1 - (1 - 0.9)(1 - 0.7)] = 0.99$$

$$R_3 = [1 - (1 - P_A)(1 - P_G)] = [1 - (1 - 0.9)(1 - 0.6)] = 0.96$$

$$R_4 = [1 - (1 - P_B)(1 - P_C)(1 - P_D)] = [1 - (1 - 0.85)(1 - 0.8)(1 - 0.75)] = 0.952$$

$$R_5 = [1 - (1 - P_B)(1 - P_C)(1 - P_E)] = [1 - (1 - 0.85)(1 - 0.8)(1 - 0.7)] = 0.951$$

$$P_6 = [1 - (1 - P_B)(1 - P_C)(1 - P_D)] = [1 - (1 - 0.85)(1 - 0.8)(1 - 0.6)]$$

$$= 0.988$$

$$P_7 = [1 - (1 - P_B)(1 - P_E)(1 - P_D)] = [1 - (1 - 0.85)(1 - 0.65)(1 - 0.6)]$$

$$= 0.979$$

$$P_8 = [1 - (1 - P_E)(1 - P_C)(1 - P_D)]$$

~~$[1 - (1 - 0.7)(1 - 0.8)(1 - 0.75)]$~~

$$= [1 - (1 - 0.7)(1 - 0.8)(1 - 0.75)] = 0.985$$

$$P_9 = [1 - (1 - P_E)(1 - P_C)(1 - P_E)] = [1 - (1 - 0.7)(1 - 0.8)(1 - 0.7)]$$

$$= 0.982$$

$$P_{10} = [1 - (1 - P_E)(1 - P_E)(1 - P_D)]$$

~~$[1 - (1 - 0.7)(1 - 0.65)(1 - 0.75)]$~~

$$P_{10} = 0.9937$$

$$P_{11} = [1 - (1 - P_E)(1 - P_E)(1 - P_E)] = [1 - (1 - 0.7)(1 - 0.65)(1 - 0.7)]$$

$$P_{11} = 0.9685$$

$$P_{12} = [1 - (1 - P_E)(1 - P_C)] = [1 - (1 - 0.7)(1 - 0.6)]$$

$$P_{12} = 0.88$$

$$P_{13} = P_1 P_2 \dots P_{12}$$

$$= (0.975)(0.97)(0.96)(0.9825)(0.991)(0.988)(0.979)$$

$$(0.985)(0.982)(0.9937)(0.9685)(0.88)$$

ଅନୁପାତ

=

0.6933

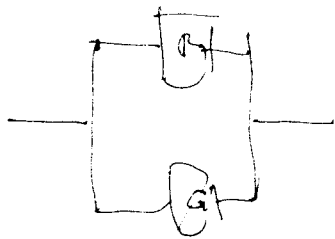
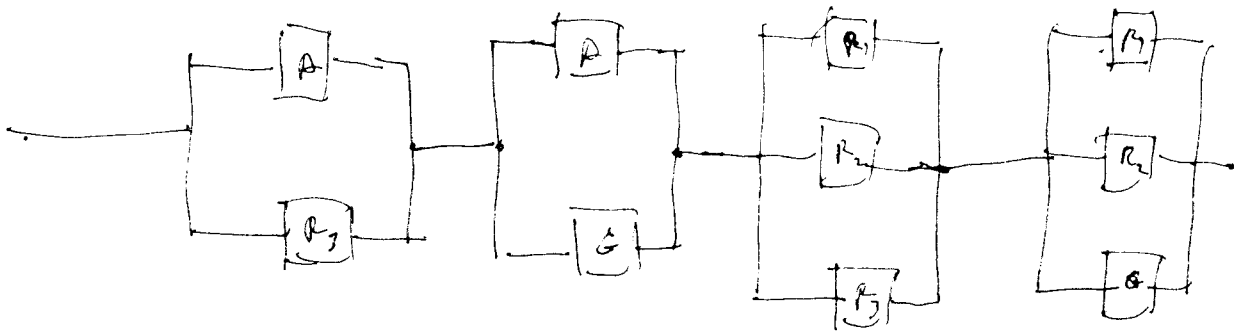
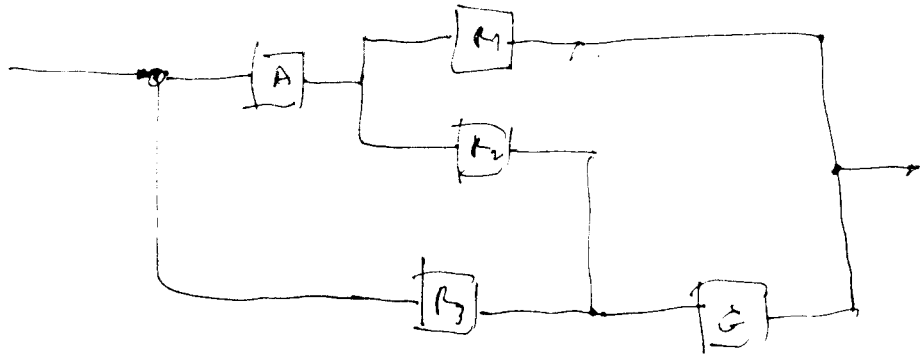
—————

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$$P_1 = P_B P_E = (0.85)(0.7) = 0.595$$

$$P_2 = P_C P_F = (0.8)(0.65) = 0.52$$

$$P_3 = P_D P_E = 0.525$$



$$\text{ကနဦးကိန်း} = [1 - (1 - P_A)(1 - P_{R_3})] [1 - (1 - P_A)(1 - P_S)] [1 - (1 - P_{R_1})(1 - P_{R_2})(1 - P_D)]$$

$$[1 - (1 - P_{R_1})(1 - P_{R_2})(1 - P_D)] [1 - (1 - P_{R_1})(1 - P_D)]$$

$$= [1 - (1 - 0.9)(1 - 0.525)] [1 - (1 - 0.9)(0.6)] [1 - (1 - 0.595)(1 - 0.52)(1 - 0.525)]$$

$$[1 - (1 - 0.595)(1 - 0.52)(1 - 0.6)] [1 - (1 - 0.595)(1 - 0.6)]$$

$$= (0.9525)(0.96)(0.9676)(0.92224)(0.874)$$

$$= 0.6838 \quad \text{--- } \text{d}$$