กลางภาค

เฉลย Math ③

ปี 2 วิศวะ โทรคม

เฉลย เฉพาะ: บท ที่ไม่เหมือนกับ วิศวโยธา

①

$$\vec{U} \times \vec{V} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 8 & 1 & 7 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 2 & -1 \\ 8 & 1 \end{matrix}$$

$$= (-7-3)\hat{i} + (24-14)\hat{j} + (2-(-8))\hat{k}$$

$$\vec{U} \times \vec{V} = -10\hat{i} + 10\hat{j} + 10\hat{k}$$

$$\vec{V} \times \vec{W} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 8 & 1 & 7 \\ 10 & 4 & 15 \end{bmatrix} \begin{matrix} \hat{i} & \hat{j} \\ 8 & 1 \\ 10 & 4 \end{matrix}$$

$$= (15-28)\hat{i} + (70-120)\hat{j} + (32-10)\hat{k}$$

$$\vec{V} \times \vec{W} = -13\hat{i} - 50\hat{j} + 22\hat{k}$$

$$(\vec{U} \times \vec{V}) \cdot (\vec{V} \times \vec{W}) = (-10\hat{i} + 10\hat{j} + 10\hat{k}) \cdot (-13\hat{i} - 50\hat{j} + 22\hat{k})$$

$$= (-10)(-13) + (10)(-50) + (10)(22)$$

$$= +130 - 500 + 220$$

$$= -150$$

๔ จงแสดงว่าเส้นตรงที่ผ่านจุด (0, 0, 5) และ (1, −1, 4)

และตั้งฉากกับเส้นตรงที่มีสมการ $\dfrac{x}{7} = \dfrac{y-3}{4} = \dfrac{z+9}{3}$

วิธีทำ Vector แสดงทิศทางของเส้นตรงที่ผ่านจุด (0, 0, 5) และ (1, −1, 4)

$$\vec{V}_1 = (1-0)\hat{i} + (-1-0)\hat{j} + (4-5)\hat{k}$$

$$\vec{V}_1 = \hat{i} - \hat{j} - \hat{k}$$ → เส้นตรง $L_1$

และ เส้นตรงที่มีสมการ

$$\frac{x}{7} = \frac{y-3}{4} = \frac{z+9}{3}$$

เทียบ form $\dfrac{x-x_0}{v_1} = \dfrac{y-y_0}{v_2} = \dfrac{z-z_0}{v_3}$

และ $\vec{V}_2 = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ → แสดงทิศทางของ

เส้นตรงที่ 2

$$\vec{V}_2 = 7\hat{i} + 4\hat{j} + 3\hat{k}$$ — เส้นตรง $L_2$

ถ้า เส้นตรง $L_1 \perp L_2$

$$\vec{V}_1 \cdot \vec{V}_2 = 0$$

$$(\hat{i} - \hat{j} - \hat{k}) \cdot (7\hat{i} + 4\hat{j} + 3\hat{k}) = 0$$

$$(1)(7)+(-1)(4)+(-1)(3)=0$$

$$7-4-3=0$$

$$0=0$$ —— เป็นจริง

แสดงว่า เส้นตรง $L_1 \perp L_2$

4.2

$$\vec{F}(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos t & \sin t & \sin t \\ t & (t+1) & t^2 \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ \cos t & \sin t \\ t & (t+1) \end{matrix}$$

$$\vec{F}'(t) = [\underbrace{t^2 \sin t}_{1} - \underbrace{(t+1)\sin t}_{2}]\,\hat{i} \;+$$

$$[\underbrace{t \sin t}_{3} - \underbrace{t^2 \cos t}_{4}]\,\hat{j} \;+$$

$$[\underbrace{(t+1)\cos t}_{5} - \underbrace{t \sin t}_{6}]\,\hat{k} \quad \longleftarrow \quad 1$$

แต่ $\vec{F}'(t)$ โดยการ Diff. ทีละพจน์

$$\frac{d}{dt} t^2 \sin t = t^2 \frac{d}{dt}\sin t + \sin t \frac{dt^2}{dt}$$

$$= t^2 \cos t + 2t \sin t \quad \longleftarrow \quad 2$$

$$\frac{d}{dt} \cancel{t^2} (t+1)\sin t = (t+1)\frac{d}{dt}\sin t + \sin t \frac{d}{dt}(t+1)$$

$$= (t+1)\cos t + \sin t \quad \longleftarrow \quad 3$$

$$\frac{d}{dt} t \sin t = t\frac{d \sin t}{dt} + \sin t \frac{dt}{dt} = t\cos t + \sin t \quad \longleftarrow \quad 4$$

$$\frac{d\,t^2\cos t}{dt} = t^2\frac{d\cos t}{dt} + \cos t\,\frac{dt^2}{dt}$$

$$= -t^2\sin t + 2t\cos t \quad \text{———} \quad (5)$$

$$\frac{d\,(t+1)\cos t}{dt} = (t+1)\frac{d}{dt}\cos t + \cos t\,\frac{d}{dt}(t+1)$$

$$= -(t+1)\sin t + \cos t \quad \text{———} \quad (6)$$

$$\frac{d\,t\sin t}{dt} = t\cos t + \sin t \quad \text{———} \quad (7)$$

แทน (2) ถึง (7) ลงใน (1)

$$\vec{F}'(t) = \left[ (t^2\cos t + 2t\sin t) - (t+1)\cos t - \sin t \right]\hat{i} +$$

$$\left[ t\cos t + \sin t + t^2\sin t - 2t\cos t \right]\hat{j} +$$

$$\left[ -(t+1)\sin t + \cos t - t\cos t - \sin t \right]\hat{k}$$

$$\vec{F}'(t) = \left[ (t^2 - t - 1)\cos t + (2t-1)\sin t \right]\hat{i} +$$

$$\left[ -t\cos t + (t^2+1)\sin t \right]\hat{j} +$$

$$\left[ -t\sin t - \sin t + \cos t - t\cos t - \sin t \right]\hat{k}$$

6

$$\bar{F}'(t) = \left[ (t^2 - t - 1)\cos t + (2t-1)\sin t \right] \hat{i} +$$

$$\left[ -t\cos t + (t^2+1)\sin t \right] \hat{j} +$$

$$\left[ (-t-2)\sin t + (1-t)\cos t \right] \hat{k}$$

5 $\quad \vec{T} = \dfrac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$

$$\hat{N} = \frac{\hat{T}'(t)}{\|\hat{T}'(t)\|} = \frac{\vec{r}''(t)}{\|\vec{r}''(t)\|}$$

$$\vec{B} = \hat{T} \times \hat{N}$$

$$\vec{r}'(t) = \underbrace{e^{7t}\cos 2t}_{①}\, \hat{i} + \underbrace{e^{7t}\sin 2t}_{②}\, \hat{j} + \underbrace{e^{7t}}_{③}\, \hat{k}$$

Diff. ทีละพจน์

$$\frac{d}{dt} e^{7t}\cos 2t = e^{7t}\frac{d\cos 2t}{dt} + \cos 2t \frac{d e^{7t}}{dt}$$

$$= -2e^{7t}\sin 2t + 7e^{7t}\cos 2t$$

$$\frac{d}{dt} e^{7t}\sin 2t = e^{7t}\frac{d\sin 2t}{dt} + \sin 2t \frac{d e^{7t}}{dt}$$

$$= 2e^{7t}\cos 2t + 7e^{7t}\sin 2t$$

$$\frac{d}{dt} e^{7t} = 7e^{7t}$$

$$\therefore \vec{r}''(t) = [-2e^{7t}\sin 2t + 7e^{7t}\cos 2t]\,\hat{i} + [2e^{7t}\cos 2t + 7e^{7t}\sin 2t]\,\cancel{\hat{k}}\,\hat{j} + 7e^{7t}\,\hat{k}$$

$$\vec{r}'\left(\frac{\pi}{2}\right) = \left[-2e^{7\frac{\pi}{2}}\overset{0}{\cancel{\sin \frac{\pi}{2}}} + 7e^{7\left(\frac{\pi}{2}\right)}\overset{-1}{\cancel{\cos \frac{\pi}{2}}}\right]\hat{i} +$$

$$\left[2e^{7\left(\frac{\pi}{2}\right)}\overset{-1}{\cancel{\cos \frac{\pi}{2}}} + 7e^{7\left(\frac{\pi}{2}\right)}\overset{0}{\cancel{\sin \frac{\pi}{2}}}\right]\hat{j} +$$

$$7e^{7\left(\frac{\pi}{2}\right)}\hat{k}$$

$$\vec{r}'\left(\frac{\pi}{2}\right) = \left(-7e^{\frac{7\pi}{2}}\right)\hat{i} + \left(-2e^{\frac{7\pi}{2}}\right)\hat{j} + 7e^{\frac{7\pi}{2}}\hat{k}$$

$$= -7e^{\frac{7\pi}{2}}\hat{i} - 2e^{\frac{7\pi}{2}}\hat{j} + 7e^{\frac{7\pi}{2}}\hat{k}$$

$$\vec{r}'\left(\frac{\pi}{2}\right) = e^{\frac{7\pi}{2}}\left(-7\hat{i} - 2\hat{j} + 7\hat{k}\right)$$

$$\left\|\vec{r}'\left(\frac{\pi}{2}\right)\right\| = \sqrt{\left(-7e^{\frac{7\pi}{2}}\right)^2 + \left(-2e^{\frac{7\pi}{2}}\right)^2 + \left(7e^{\frac{7\pi}{2}}\right)^2}$$

$$= \sqrt{49e^{7\pi} + 4e^{7\pi} + 49e^{7\pi}}$$

$$= \sqrt{102}\, e^{\frac{7\pi}{2}}$$

$$\hat{T}\left(\frac{\pi}{2}\right) = \frac{\vec{r}'\left(\frac{\pi}{2}\right)}{\left\|\vec{r}'\left(\frac{\pi}{2}\right)\right\|}$$

$$\hat{T}\left(\frac{\pi}{2}\right) = \frac{-7e^{7\frac{\pi}{2}}\hat{i} - 2e^{7\frac{\pi}{2}}\hat{j} + 7e^{7\frac{\pi}{2}}\hat{k}}{\sqrt{102}\, e^{7\frac{\pi}{2}}}$$

$$\hat{T}\left(\frac{\pi}{2}\right) = -\frac{7}{\sqrt{102}}\hat{i} - \frac{2}{\sqrt{102}}\hat{j} + \frac{7}{\sqrt{102}}\hat{k} \quad \text{---} \quad (4)$$

(b) $\displaystyle\int_{-1}^{1} \left[ (1+t)^{\frac{3}{2}} \hat{i} + (1-t)^{\frac{3}{2}} \hat{j} \right] dt$

$$= \int_{-1}^{1} (1+t)^{\frac{3}{2}}\, dt + \int_{-1}^{1} (1-t)^{\frac{3}{2}}\, dt$$

$$= \left. \frac{(1+t)^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right|_{-1}^{1} + \frac{(1-t)^{\frac{3}{2}+1}}{\frac{3}{2}+1}$$

$$= \left. \frac{(1+t)^{\frac{5}{2}}}{\frac{5}{2}} \right|_{-1}^{1} + \left. \frac{(1-t)^{\frac{5}{2}}}{\frac{5}{2}} \right|_{-1}^{1}$$

$$= \left. \frac{2}{5}(1+t)^{\frac{5}{2}} \right|_{-1}^{1} + \left. \frac{2}{5}(1-t)^{\frac{5}{2}} \right|_{-1}^{1}$$

$$\frac{2}{5}\left[(2)^{\frac{5}{2}} - 0\right] + \frac{2}{5}\left[0 - (2)^{\frac{5}{2}}\right]$$

$$\cancel{= \frac{2}{5}\left[\left(2^{\frac{5}{2}} - 0\right) + \left(0 - 2^{\frac{5}{2}}\right)\right]}$$

$$= \frac{2}{5}\, 2^{\frac{5}{2}} - \frac{2}{5}\,(2)^{\frac{5}{2}} \cancel{= 0}$$

$$\cancel{= \frac{2}{5}\left(2^{\frac{5}{2}} - 2^{\frac{5}{2}}\right) = 0}$$

$$= \frac{2}{5}(2)^{\frac{5}{2}}\hat{i} - \frac{2}{5}(2)^{\frac{5}{2}}\hat{j}$$

8) จงหา $\nabla\phi$ เมื่อ $\phi(x,y,z) = 4xz^3 - 3x^2y^2z$ ที่จุด $(2,1,-1)$

$$\nabla\phi = \frac{\partial}{\partial x}(4xz^3 - 3x^2y^2z)\,\hat{i} +$$

$$\frac{\partial}{\partial y}(4xz^3 - 3x^2y^2z)\,\hat{j} +$$

$$\frac{\partial}{\partial z}(4xz^3 - 3x^2y^2z)\,\hat{k}$$

$$\nabla\phi = (4z^3 - 6y^2zx)\,\hat{i} +$$

$$(0 - 6x^2zy)\,\hat{j} +$$

$$(12xz^2 - 3x^2y^2)\,\hat{k}$$

$$\nabla\phi(2,1,-1) = [4(-1)^3 - 6(1)^2(-1)(2)]\,\hat{i} +$$

$$[-6(2)^2(-1)(1)]\,\hat{j} +$$

$$[12(2)(-1)^2 - 3(2)^2(1)^2]\,\hat{k}$$

$$\nabla\phi(2,1,-1) = 8\hat{i} + 24\hat{j} + 12\hat{k}$$