

ԴՐԱՄԱ

ՀԱՇՎ ՄԱԹ ③

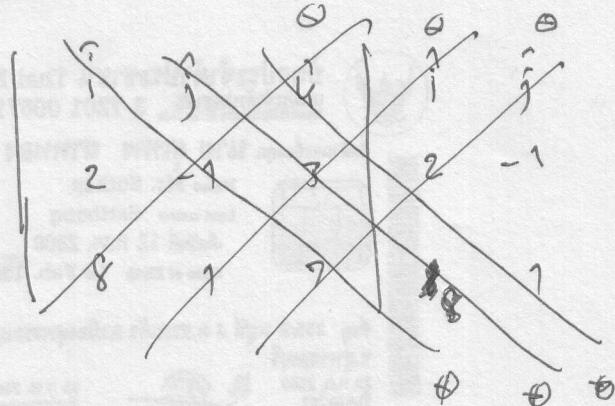
\tilde{U}_2 գովազարար

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①

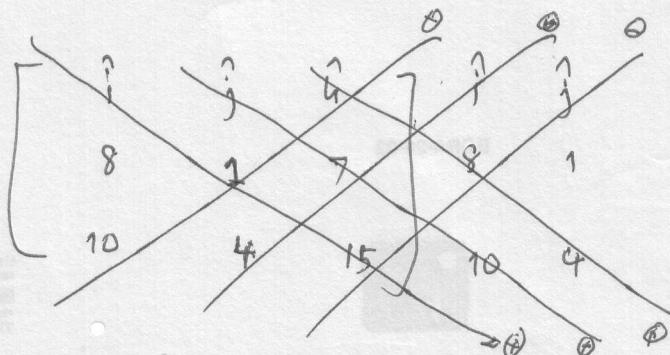
$$\tilde{U} \times \tilde{V} =$$



$$= (-7 - 8) \hat{i} + (24 - 4) \hat{j} + (2 - (-8)) \hat{k}$$

$$\tilde{U} \times \tilde{V} = -10 \hat{i} + 10 \hat{j} + 10 \hat{k}$$

$$\tilde{V} \times \tilde{W} =$$



$$= (15 - 28) \hat{i} + (70 - 120) \hat{j} + (32 - 10) \hat{k}$$

$$\tilde{V} \times \tilde{W} = -13 \hat{i} - 50 \hat{j} + 22 \hat{k}$$

$$(\tilde{U} \times \tilde{V}) \cdot (\tilde{V} \times \tilde{W}) = (-10 \hat{i} + 10 \hat{j} + 10 \hat{k}) \cdot (-13 \hat{i} - 50 \hat{j} + 22 \hat{k})$$

$$= (-10)(-13) + (10)(-50) + (10)(22)$$

$$= +130 - 500 + 220$$

$$= -150$$

12

(2) $\text{co-ordinates of intersection point } (0, 0, 5) \text{ l.e. } (1, -1, 4)$

(2)

$$\text{Ratio form of line equation of line. } \frac{x}{7} = \frac{y-3}{4} = \frac{z+9}{3}$$

S. I.

Vector equation of line passing through points $(0, 0, 5)$ & $(1, -1, 4)$

$(1, -1, 4)$

$$\vec{v}_1 = (1-0)\hat{i} + (-1-0)\hat{j} + (4-5)\hat{k}$$

$$\vec{v}_1 = \hat{i} - \hat{j} - \hat{k} \rightarrow \text{Direction L}_1$$

Now find direction ratios

$$\frac{x}{7} = \frac{y-3}{4} = \frac{z+9}{3}$$

Now form

$$\frac{x-x_0}{v_1} = \frac{y-y_0}{v_2} = \frac{z-z_0}{v_3}$$

$$\text{Now } \vec{v}_2 = v_1\hat{i} + v_2\hat{j} + v_3\hat{k} \rightarrow \text{Direction L}_2$$

Direction L₂

$$\vec{v}_2 = 7\hat{i} + 4\hat{j} + 3\hat{k} \rightarrow \text{Direction L}_2$$

As lines L₁ \perp L₂

$$\vec{v}_1 \cdot \vec{v}_2 = 0$$

$$(\hat{i} - \hat{j} - \hat{k}) \cdot (7\hat{i} + 4\hat{j} + 3\hat{k}) = 0$$

③

$$(1)(-7) + (-1)(4) + (-1)(3) = 0$$

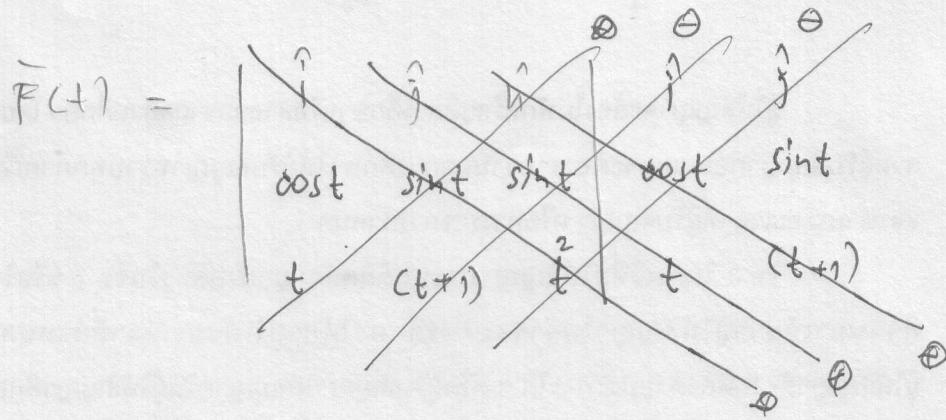
$$-7 - 4 - 3 = 0$$

$$0 = 0 \quad \text{_____} \quad \text{Ans 75b}$$

1860's
about $L_1 \perp L_2$

b

Q.2



$$\begin{aligned} \tilde{F}'(t) &= \left[\underbrace{t^2 \sin t}_{\textcircled{1}} - \underbrace{(t+1) \sin t}_{\textcircled{2}} \right] \hat{j} + \\ &\quad \left[\underbrace{t \sin t}_{\textcircled{3}} - \underbrace{t^2 \cos t}_{\textcircled{4}} \right] \hat{i} + \\ &\quad \left[\underbrace{(t+1) \cos t}_{\textcircled{5}} - \underbrace{t \sin t}_{\textcircled{6}} \right] \hat{k} \quad \longleftarrow \textcircled{1} \end{aligned}$$

an $\tilde{F}'(t)$ fann Diff. n. n. n.

$$\begin{aligned} \frac{d}{dt} t^2 \sin t &= t^2 \frac{d \sin t}{dt} + \sin t \frac{d t^2}{dt} \\ &= t^2 \cos t + 2t \sin t \quad \longleftarrow \textcircled{2} \end{aligned}$$

$$\begin{aligned} \frac{d}{dt} (t+1) \sin t &= (t+1) \frac{d \sin t}{dt} + \sin t \frac{d(t+1)}{dt} \\ &= (t+1) \cos t + \sin t \quad \longleftarrow \textcircled{3} \end{aligned}$$

$$\frac{d}{dt} t \sin t = t \frac{d \sin t}{dt} + \sin t \frac{d t}{dt} = t \cos t + \sin t \quad \textcircled{4}$$

$$\begin{aligned}\frac{d^2 \cos t}{dt^2} &= t^2 \frac{d \cos t}{dt} + \cos t \frac{d t^2}{dt} \\ &= -t^2 \sin t + 2t \cos t \quad \text{--- (5)}\end{aligned}$$

$$\begin{aligned}\frac{d(t+1) \cos t}{dt} &= (t+1) \frac{d \cos t}{dt} + \cos t \frac{d(t+1)}{dt} \\ &= -(t+1) \sin t + \cos t \quad \text{--- (6)}\end{aligned}$$

$$\frac{d t \sin t}{dt} = t \cos t + \sin t \quad \text{--- (7)}$$

由 (2) 及 (7) 索以 (1)

$$\bar{F}'(t) = [(t^2 \cos t + 2t \sin t) - (t+1) \cos t - \sin t] i +$$

$$[t \cos t + \sin t + t^2 \sin t - 2t \cos t] j +$$

$$[-(t+1) \sin t + \cos t - t \cos t - \sin t] k$$

$$\bar{F}'(t) = [(t^2 - t - 1) \cos t + (2t - 1) \sin t] i +$$

$$[-t \cos t + (t^2 + 1) \sin t] j +$$

$$[-t \sin t - \sin t + \cos t - t \cos t - \sin t] k$$

(1) 12

$$\bar{F}^{(t+1)} = \left[(t^2 - t - 1) \cos t + (2t - 1) \sin t \right] \hat{i} +$$

$$\left[-t \cos t + (t^2 + 1) \sin t \right] \hat{j} +$$

$$\left[(-t - 2) \sin t + (1 - t) \cos t \right] \hat{k}$$

⑨

$$\hat{\tau} = \frac{\vec{\gamma}'(t)}{\|\vec{\gamma}'(t)\|}$$

$$\hat{n} = \frac{\hat{\tau}'(t)}{\|\hat{\tau}'(t)\|} = \frac{\vec{\gamma}''(t)}{\|\vec{\gamma}''(t)\|}$$

$$\hat{B} = \hat{\tau} \times \hat{n}$$

$$\vec{\gamma}'(t) = \underbrace{e^t \cos 2t}_{\textcircled{1}} \hat{i} + \underbrace{e^t \sin 2t}_{\textcircled{2}} \hat{j} + \underbrace{e^t \hat{u}}_{\textcircled{3}}$$

Diff. ဆုံးမျှ

$$\frac{d}{dt} e^t \cos 2t = e^t \frac{d \cos 2t}{dt} + \cos 2t \frac{de^t}{dt}$$

$$= -2e^t \sin 2t + 2e^t \cos 2t$$

$$\frac{d}{dt} e^t \sin 2t = e^t \frac{d \sin 2t}{dt} + \sin 2t \frac{de^t}{dt}$$

$$= 2e^t \cos 2t + 2e^t \sin 2t$$

$$\frac{d}{dt} e^t = 2e^t$$

$$\vec{\gamma}'(t) = [-2e^t \sin 2t + 2e^t \cos 2t] \hat{i} + [2e^t \cos 2t + 2e^t \sin 2t] \hat{j} + 2e^t \hat{k}$$

$$\begin{aligned}\tilde{\gamma}'\left(\frac{\pi}{2}\right) &= \left[-2 e^{\frac{7\pi}{2}} \cancel{\sin \frac{\pi}{2}} + 7 e^{\frac{7\pi}{2}} \cancel{\cos \frac{\pi}{2}} \right]^{-1} \hat{i} + \\ &\quad \left[2 e^{\frac{7\pi}{2}} \cancel{\cos \frac{\pi}{2}}^{-1} + 7 e^{\frac{7\pi}{2}} \cancel{\sin \frac{\pi}{2}} \right] \hat{j} + \\ &\quad 7 e^{\frac{7\pi}{2}} \hat{k}\end{aligned}$$

$$\tilde{\gamma}'\left(\frac{\pi}{2}\right) = (-7 e^{\frac{7\pi}{2}}) \hat{i} + (-2 e^{\frac{7\pi}{2}}) \hat{j} + 7 e^{\frac{7\pi}{2}} \hat{k}$$

$$= -7 e^{\frac{7\pi}{2}} \hat{i} - 2 e^{\frac{7\pi}{2}} \hat{j} + 7 e^{\frac{7\pi}{2}} \hat{k}$$

$$\tilde{\gamma}'\left(\frac{\pi}{2}\right) = e^{\frac{7\pi}{2}} (-7 \hat{i} - 2 \hat{j} + 1 \hat{k}) \quad \cancel{-}$$

$$\|\tilde{\gamma}'\left(\frac{\pi}{2}\right)\| = \sqrt{(-7 e^{\frac{7\pi}{2}})^2 + (-2 e^{\frac{7\pi}{2}})^2 + (7 e^{\frac{7\pi}{2}})^2}$$

$$= \sqrt{49 e^{7\pi} + 4 e^{7\pi} + 49 e^{7\pi}}$$

$$= \sqrt{102} e^{\frac{7\pi}{2}}$$

$$\text{答 } \hat{T}\left(\frac{\pi}{2}\right) = \frac{\tilde{\gamma}'\left(\frac{\pi}{2}\right)}{\|\tilde{\gamma}'\left(\frac{\pi}{2}\right)\|} *$$

⑤

$$\hat{T}\left(\frac{\pi}{2}\right) = \frac{-\sqrt{2}e^{\frac{i\pi}{2}}\hat{i} - 2e^{\frac{3\pi}{2}}\hat{j} + \sqrt{2}e^{\frac{5\pi}{2}}\hat{k}}{\sqrt{102}}$$

$$\hat{T}\left(\frac{\pi}{2}\right) = -\frac{\sqrt{2}}{\sqrt{102}}\hat{i} - \frac{2}{\sqrt{102}}\hat{j} + \frac{\sqrt{2}}{\sqrt{102}}\hat{k} - \cancel{B}$$

10

$$\textcircled{6} \quad \int_{-1}^1 \left[(1+t)^{\frac{3}{2}} \mathbf{i} + (1-t)^{\frac{3}{2}} \mathbf{j} \right] dt$$

$$= \int_{-1}^1 (1+t)^{\frac{3}{2}} dt + \int_{-1}^1 (1-t)^{\frac{3}{2}} dt$$

$$= \left. \frac{(1+t)^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right|_{-1}^1 + \left. \frac{(1-t)^{\frac{3}{2}+1}}{\frac{3}{2}+1} \right|_{-1}^1$$

$$= \left. \frac{(1+t)^{\frac{5}{2}}}{\frac{5}{2}} \right|_{-1}^1 + \left. \frac{(1-t)^{\frac{5}{2}}}{\frac{5}{2}} \right|_{-1}^1$$

$$= \frac{2}{5} (1+t)^{\frac{5}{2}} \Big|_{-1}^1 + \frac{2}{5} (1-t)^{\frac{5}{2}} \Big|_{-1}^1$$

$$= \frac{2}{5} \left[(2)^{\frac{5}{2}} - 0 \right] + \frac{2}{5} \left[0 - (2)^{\frac{5}{2}} \right]$$
~~$$= \frac{2}{5} \left[(2)^{\frac{5}{2}} - 0 \right] + \frac{2}{5} \left[0 - (2)^{\frac{5}{2}} \right]$$~~

~~$$= \frac{2}{5} \left(2^{\frac{5}{2}} - 2^{\frac{5}{2}} \right)$$~~

~~$$= \frac{2}{5} (2)^{\frac{5}{2}} \mathbf{i} - \frac{2}{5} (2)^{\frac{5}{2}} \mathbf{j}$$~~

$$⑧ \quad \text{so} \quad \nabla \phi \quad \text{is} \quad \phi(x, y, z) = 4x^3 - 3x^2y^2z \quad \text{at } (2, 1, -1)$$

$$\nabla \phi = \frac{\partial}{\partial x} (4x^3 - 3x^2y^2z) \hat{i} +$$

$$\frac{\partial}{\partial y} (4x^3 - 3x^2y^2z) \hat{j} +$$

$$\frac{\partial}{\partial z} (4x^3 - 3x^2y^2z) \hat{k}$$

$$\nabla \phi = (4x^3 - 6y^2z) \hat{i} +$$

$$(0 - 6x^2yz) \hat{j} +$$

$$(12x^2 - 3x^2y^2) \hat{k}$$

$$\nabla \phi(2, 1, -1) = [4(-1)^3 - 6(1)^2(-1)(2)] \hat{i} +$$

$$[-6(2)^2(-1)(1)] \hat{j} +$$

$$[12(2)(-1)^2 - 3(2)^2(1)^2] \hat{k}$$

$$\nabla \phi(2, 1, -1) = 8 \hat{i} + 24 \hat{j} + 12 \hat{k} \quad \text{--- } \textcircled{2}$$