Recent Advancements in Forward Error Correcting Channel Codes

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Abstract

Channel codes are used to improve the transmission quality of the message when it faces disturbances like noise interference, Doppler shift etc. in a shared media, where multiple users are transmitting data simultaneously, channel codes are needed to guarantee quality of service during transmission. In this paper, recent advancements in channel coding have been discussed. Starting from channel coding history when Shannon gave the theoretical transmission capacity limits to the state of the art Turbo codes, Low-Density Parity-Check (LDPC) codes and Polar codes. A comparison has been done on the latest forward error correcting (FEC) codes.

Keywords: Belief propagation, channel coding, LDPC codes, Polar codes

1. Introduction

The journey of error-correcting codes started from the famous piece of work by Shannon in Bell Systems Technical Journal in 1949 [1]. At that time, there were some advancements in various methods of modulation such as pulse code modulation (PCM) and pulse position modulation (PPM) which exchange bandwidth for signal to noise ratio. Fundamentals of the general theory of communication were built by Nyquist [2] and Hartley [3] in 1924 and 1928 respectively. The renowned letter to the Editor from Marcel Golay, notes on digital coding which highlights the ways by which channel/message coding can be used as a mean to attain a theoretical capacity of a communication channel by reducing the probabilities of error. Another great scientist from Bell Labs named Richard Hamming introduced his own codes known as Hamming codes [4]. These codes belong to the family of linear error correcting codes and have the capability to detect up to two bit errors or correct up to one bit error without detecting them. At that time, Hamming codes were considered far superior as parity codes can only detect the errors and don't have the ability to correct them.

The time when the research on linear block codes was to rise, Peter Elias invented a new domain of error correcting codes named as convolutional codes [5]. After the invention of convolutional codes in 1955, another significant contribution was made by Irving S. Reed and Gustave Solomon in 1960 with their own error correcting codes known as Reed Solomon codes [6]. The same time, a low-density parity-check (LDPC) code which belongs to the class of linear error correcting code were invented by R. Gallagar during his PhD Thesis at Massachusetts Institute of Technology. At the time, the complexity of LDPC codes was considered to be high and these codes were ignored for almost thirty years.

Three decades later, after the invention of LDPC codes, Turbo codes were introduced which are considered to be the first high performance forward error correcting (FEC) codes [7]. These codes have superior performance than the preciously invented codes and it were able to achieve the channel capacity limit to some extent. Because of the practical nature of the Turbo codes, these codes were adopted as a standard in 3^{rd} and 4^{th} Generation (3G/4G) mobile communication. After the invention of Turbo codes, race in the field of error correcting codes started and LDPC codes were re-discovered by Neal and Mackay in 1998 [8]. The re-discovered LDPC codes have a comparable performance with the Turbo codes and currently both codes are used as workhorses in modern digital communication world.

The roots of LDPC codes have a resemblance to random coding. It has been observed that a randomly generated code with optimal decoder has, in general, good performance. It was Gallagar vision and he believes that these random codes can be decoded effectively by an iterative algorithm commonly known as belief propagation (BP) decoding. The asymptotic performance of an iterative decoder can be viewed by observing the probability distribution of exchanged messages in the Tanner graph. When the degree of the nodes in the Tanner graph is optimized, it results in irregular LDPC codes yielding better belief propagation thresholds towards capacity. Convolutional LDPC codes were invented by Jimenez Feltstrom *et. al.*[11]. These codes also have sparse parity check matrices with a band-diagonal structure. When the performance of LDPC conventional codes over BP was observed, it can be seen that the performance was significantly improved by spatial coupling. The state of the art research in the field of LDPC suggests that if non-binary LDPC codes are decoded in frequency domain, a significantly reduced complex transform can be designed.

This paper has been organized as follows. Section 2 discusses the Polar codes, Section 3 talks about the comparison of modest FEC channel codes and the paper concludes in Section 4.

2. Polar Codes

One of the major benchmark achievement in the first decade of the 21st century is the origin of Polar codes in channel coding theory. This historical work was led by E. Arikan [12]. Earlier, this work started with the targets of channel capacity approaching techniques based on polarization of information and further decoding based on successive-cancelation framework in convolutional decoder. Polar codes can achieve capacity having reduced decoding complexity of $O(N\log_2 N)$,where $N$is codeword length[12]. Originally, polar codes intend to be concatenated inner code scheme with convolutional outer codes. However, at the end, polar codes become so powerful that there is no need of the outer code with frame error rate probability intend to zero, roughly as $e^{-\sqrt{N}}$ for any fixed rate below capacity [13].

Various articles since 2009 on Polar codes investigates different aspects in terms of design of Polar codes, optimized decoding techniques, and extensions have been produced. Very recently 3GPP proposed polar codes as channel code to be used in control channel on 5G standard, along with data channel using LDPC codes for high throughput and error performance. Currently, the only existing codes in forward error correcting techniques (FEC), which can asymptotically achieve the capacity of binary input symmetric discrete memoryless channel for ideally infinite block length with unambiguous construction. Moreover, their obvious potential application in various wireless communication and signal processing need to be addressed and enormous area of investigation and analysis lies in the research community. In support of its promising results various categories of Polar codes proposed for different applications such as generalized polar codes [14], compound polar codes [15], concatenated polar codes [16], and universal polar codes [17].

2.1 Encoding

In general, basic binary polar code is a linear block code defined for any block length $N = 2^n$ and $n = \log_2(N)$, the generator matrix and kernel F can be represented as

$$G_N = F^{\otimes n}, F = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \tag{1}$$

The encoder is basically the polarization transform given by a kernel [12]. This transform for a higher input size can be obtained by *Kronecker* product of F with itself, denoted as $F^{\otimes n}$, causing block length N. The encoded codeword generated in this case, polar code (8,4) as follows

$$\mathbf{G_8} = \begin{bmatrix} \boxed{\begin{matrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}} & \boxed{\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{matrix}} \\ \boxed{\begin{matrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}} & \boxed{\begin{matrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}} \end{bmatrix} \tag{2}$$

In the next step, freezing or unreliable bits need to be accomplished in such a way so that designed polar codes give the best performance under SC decoding in a given channel. Let us take an example of freezing bits $[u_1, u_2, u_5, u_8]$ fixed to be zero. Thereafter, one can obtain **G** as follows:

$$\mathbf{G} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{3}$$

The encoder part is primarily channel dependent, therefore no specific frozen bit condition so far that will uniquely work for all the channels [18-25].

Furthermore, polar code work extended to non-binary alphabets in [26-28].

2.2 Decoding

Polar code can be decoded using commonly used belief propagation algorithm, still the standard decoding algorithm is SC based decoder. In SC based decoder, the error performance is not up to the mark due to suboptimal nature and week minimum distance properties of SC algorithm. In this algorithm, decision made sequentially, therefore decoder latency will increase in large block length and causes throughput blockage. Instead of several shortcomings, polar codes interest in a number of potential application continued. Moreover, in early stages of development, BP decoding [29] and maximum likelihood (ML) decoding [30] studied to improve the performance of polar codes, however, BP based decoding does not work to improve error performance by a significant amount but convincing to achieve higher throughput as compared to SC.

In addition, list decoder investigated as in [19]. Interestingly, further notable significant improvement can be achieved by using list decoder with CRC, which can have achieved near ML performance with less complexity roughly $O(N \log_2 N)$ for a list size of N and code length L. CRC helps in coding efficiency in terms of code performance for high SNR by restricting minimum distance and it also helps to select the correct code word from the set of combination of offered code words by using the list decoder [31].

2.3 Performance of Polar codes

We examine here the performance of systematic polar codes using standard SC based decoder. The results are given for (1024, 512) codes for half code rate. Frame error rate and bit error rate are shown in Fig. 1 and Fig. 2 respectively.

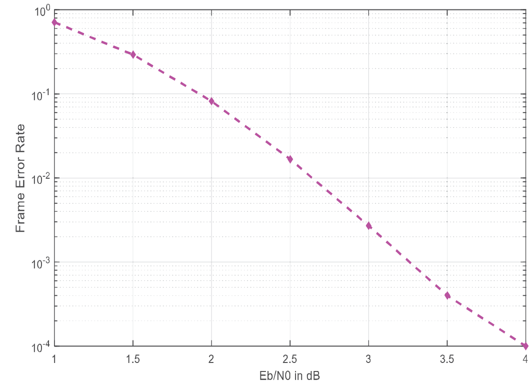

Figure 1: FER Systematic Polar code performance

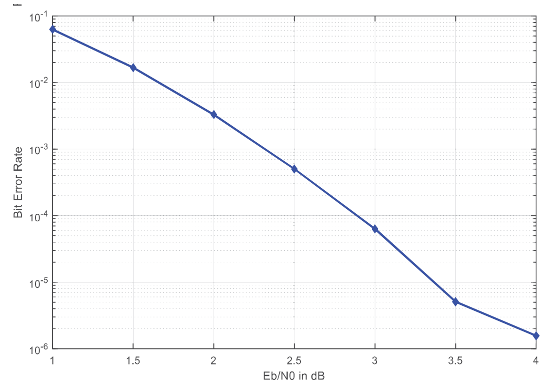

Figure 2: BER Systematic Polar code performance

3. Comparison of Modest FEC Channel Codes

The three codes are compared over some of the important parameters so as to get an overall idea of how efficient a code is. The Table 1 compares three codes viz. Turbo, LDPC and Polar over certain parameters.

Table 1: FEC Channel codes comparision

Parameters	FEC Channel Codes		
	Turbo code	LDPC	Polar
Year of invention	1993 [7]	1960 [33]	2008 [12]
Proposed researcher	Berrou, Glavieux, and Thitimajshima	Gallager	Arikan
Shannon limit performance.	0.03db [34]	0.0045db [35]	Capacity approaching [12]
Encoding [36]	Two recursive convolutional encoders	Sparse parity check matrix	Generator matrix using channel polarization
Error correction capability [37]	Similar	Similar	Similar for list decoding +CRC
Decoding [36]	Trellis termination or Viterbi decoder.	Layered belief propagation and sum-product algorithm	List decoders and CRC
Hardware Compatibility [37]	High	High	Low

Throughput (Gbps) [37]	21.9	78	208
Processing Latency(μs) [2]	0.24 K=6144 R=1/3	0.06 K=1723 R=0.84	3.21 K=1024 R=1/2
Complexity [37]	Higher for most coding rates	Lower for most coding rates	Lower for most coding rates
Flexibility [37]	Flexible	Inflexible	Inflexible
Backward compatibility[37]	Yes	No	No
Maturity [37]	High Proven in 3G/4G	High Wi-Fi	Limited Unproven
Hardware Efficiency [Mbps/mm^2][37]	279	158	402
Computational complexity [MaxMinAdd operations per data per bit]	4340 Irrespective of block length and rate [38]	$\left(6\ mean\left[d_v\right] - 9\right)/R + 6$ [36]	$log_2\ \left(K/R\right)/R$ [12]
Decoding complexity	$O\left(I_{max}\left(4N2^m\right)\right)$ BCJR [7]	$O\left(I_{max}\left(N\overline{d_v} + M\ \overline{d_c}\right)\right)$ BP [9]	$O(N \log N)$ SC [12]
High-performance flexible implementation complexity [37]	Lower	Higher	Unproven
Interconnect complexity [37]	Lower	Higher	Lower

4. Conclusion

In this paper, we studied advancements of modern channel coding and its challenges. We have summarized polar codes and provide a relative comparison of Turbo codes, LDPC codes and polar codes. With certain exception, polar codes seem to have promising performance, but at the same time it is very much channel dependent codes. There are still scope of research left, especially in non-binary LDPC and polar codes for optimal decoding and maximum throughput. In current scenario, it is noticed that researches interest shift to spatially coupled LDPC and polar codes to solve issues for future coding standard problems. We hope to show research community that is coding theory is still an active research area with many challenges remaining.

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