Combined Coding Algorithm of LDPC Codes in Bit-Patterned Media Recording Channels

Chutima Prasartkaew[1,a], Piya Kovintavewat[2,b] and Somsak Choomchuay[3,c]

[1]Department of Mathematics and Computer Science, Faculty of Science and Technology,

Rajamangala University of Technology Thunyaburi, Thailand

[2]Data Storage Technology Research Center, NakhonPathomRajabhat University, Thailand

[3]Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Thailand

[a]chutima_p@rmutt.ac.th, [b]piya@npru.ac.th, [c]kchsomsa@kmitl.ac.th

Keywords: Bit-patterned media recording, irregular LDPC codes, magic square based algorithm, random number table.

Abstract. This paper presents a new coding algorithm for irregular low-density parity-check (LDPC) codes, which consists of two parts within one block code, namely the data bits and the parity bits. Specifically, the data bits are encoded using a random number table, whereas the parity bits will be further encoded based on a magic square based algorithm (MSBA). We refer to this proposed coding algorithm as the combined coding algorithm for LDPC codes (CCA-LDPC codes), which can be used in bit-patterned media recording (BPMR) channels. The performance of the proposed codes is numerically simulated in a BPMR channel model, and compares with that of the existing LDPC codes. Simulation results indicate that, at a block length of 4110 bit and code rate of 0.9, the proposed CCA-LDPC codes yields better performance than the existing LDPC codes at high signal-to-noise ratio (SNR) scenario.

Introduction

There are many types of error-correcting codes, which have been used in communication and data recording systems. One of the most popular codes and currently being used in such systems is an LDPC code. In general, the LDPC code is a linear block code defined by a sparse parity check matrix, whose parity check matrix was randomly generated [1]. Then, the LDPC code was re-tested by Mackay [2], which showed that the code performance can be made close to the Shannon's limit. Therefore, many researchers were interested in LDPC codes. In addition, Mackay pointed out that the decoding complexity of the sum product algorithm is linear, meaning that there is a constraint for the code with long block length. Furthermore, an array code was proposed by Blaum *et al*. [3], which has the capability to correct an error burst using an algebraic decoder, but can also be fully incorporated into a soft iterative decoding scheme with an algebraic decoder. The symbols for array codes can be large, thus making it suitable for correcting a long burst of errors.

Prasartkeaw and Choomchuay [4] introduced a new construction algorithm for the short block irregular LDPC code. By applying a magic square theorem as a part of the matrix construction, a newly developed algorithm denoted as MSBA is obtained. Simulation result based on an additive white Gaussian noise (AWGN) channel shows that at a code rate of 0.8 and SNR of 5 dB, the BER of 10^{-7} can be obtained whilst the number of decoding iterations is relatively low. Additionally, Prasartkaew *et al.* [5] also presented the application of a random number table for the parity check matrix construction of irregular LDPC codes, and tested the code performance in a BPMR channel. This paper proposes a new algorithm for coding scheme referred to as the CCA-LDPC code.

Magic Square Based Algorithm

A magic square is a square array of the numbers 1, 2, 3, ..., z^2, with the property that the summation of every row, column, and both diagonals, is the same number. Since there are z rows and z columns, the summation of all the numbers in the magic square must be $z\times M$, where M is the number that each row, column, and diagonal must add up to, which must satisfy

$$\sum_{i=1}^{z^2} i = z \times M \tag{1}$$

Then, solving for M gives $M = z(z^2 + 1)/2$. According to how they are generated, the existing magic square can be classified into four groups [4], i.e., the magic square associated with (a) astrological planets, (b) odd order, (c) doubly even order, and (d) singly even order. Among these, the best codes are obtained by using a 6×6 magic square of group (d) [4]. Consequently, the magic square generated by Strachey-method (the best one in group (d)) will be used in this work.

Random Number Table Based Algorithm

Random number tables have been used in statistics for tasks such as selected random samples. Tables of random numbers have the desired properties, no matter how the required numbers are chosen from the table: by row, column, diagonal, or irregularly. The first table was published by L.H.C. Tippett in 1927 [5]. To select the required numbers from the random numbers, we could choose all required numbers at any point of the table for required-digit number (e.g.: one-, two-, or three-digits). Then, keep going until we have enough different numbers (two-digit for this study) for filling in the sub-matrix as a shifting order. The parity check matrix, **H**, can be constructed according to the following steps [5].

Step 1. Define the designed parameters of J, K and L to be an integer greater than 3, where $\{J, K\} \leq L$. All values must satisfy the following condition:

$$\left(J \times K\right) - K - \lambda \leq L, \quad \lambda = 5, 9, 14, \ldots \tag{2}$$

Step 2. Place two-digit random number in the structured parity check matrix **H**. These two-digit random numbers will be used as a shifting order of the circulant matrix of $L \times L$.

BPMR Channel

A typical discrete-time BPMR channel model is illustrated in Fig. 1, where we assume only two adjacent tracks cause most of the ITI. Therefore, the read-back signal can be expressed as

$$y_k = \sum_i \sum_m h_{i,m} u_{k-i,m} + n_k$$
(3)

where $u_{k,0}$, $u_{k,-1}$, and $u_{k,1} \in \{\pm 1\}$ represent an independent and identically distributed (*i.i.d.*) binary input bit sequences with bit period T in the main track and the two adjacent tracks, respectively, $h_{i,m}$'s are the 2D channel response coefficients, and n_k is AWGN with zero mean and variance σ^2. Without the track mis-registration, we consider a discrete-time 3-by-3 symmetric channel response matrix of the form

$$H = \begin{bmatrix} h_{0,-1} & h_{1,-1} & h_{2,-1} \\ h_{0,0} & h_{1,0} & h_{2,0} \\ h_{0,1} & h_{1,1} & h_{2,1} \end{bmatrix} = \begin{bmatrix} 0.0347 & 0.2297 & 0.0347 \\ 0.1277 & 1 & 0.1277 \\ 0.0347 & 0.2297 & 0.0347 \end{bmatrix}$$
(4)

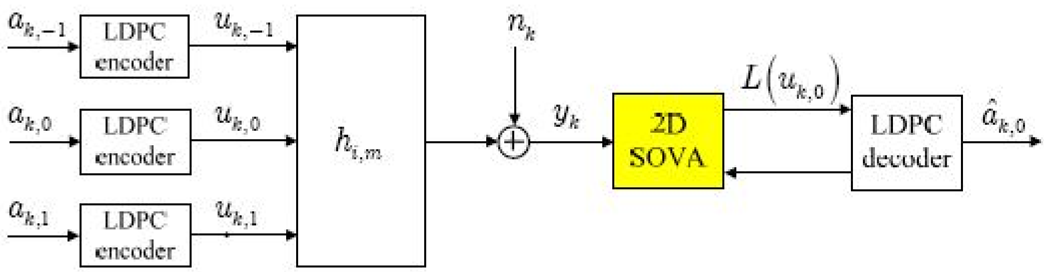

Fig. 1. A typical discrete-time BPMR channel model.

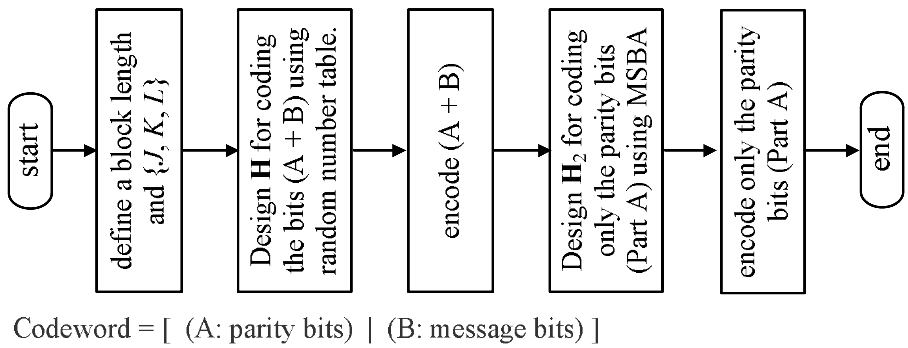

Fig. 2. Flowchart of the proposed coding algorithm.

which is for a magnetic medium with SUL [7]. Accordingly, the sequence y_k is sent to a turbo equalizer, which iteratively exchanges soft information between the proposed two-dimensional (2D)
SOVA equalizer and the LPDC decoder implemented based on the message passing algorithm with 3 internal iterations. To perform maximum-likelihood equalization via the 2D SOVA, we propose to use a similar technique that was employed in the so-called bidirectional SOVA [8] to compute the LLR of the bit $u_{k,0}$, i.e., $L(u_{k,0})$. For this 3-by-3 channel matrix, the trellis of this 2D SOVA will have 36 states with six parallel branches between any two connected states.

Combined Coding Algorithm

This paper proposed a new coding scheme, which is a combined method between the methods presented in [4] (referred to as MSBA) and [5] (referred to as TRN). Let the codeword consist of two parts, namely, the parity bits (Part A) and the message bits (Part B). Then, Fig. 2 shows the flowchart of the proposed coding scheme.

Simulation Results

The performance of the proposed codes was tested via simulation, and the obtained results are compared with the existing codes, where their parity check matrices are generated based on random number table (denoted as "Tippett's Random") [5] and based on Random Gallager Codes (referred to as "MacKay") [6]. The parameters used in our simulation are: $J = 3$, $K = 30$, and $L = 137$ such that all codes will have same block length of 4110 bits, the parity bits of $J \times L = 411$, the message bits of $K \times L = 3699$, and the code rate of $R = 1 - J/K = 0.9$. The SNR is defined as E_b/N_0 in decibel (dB), where E_b is the energy per bit and $N_0 = (2T)\sigma^2$.

Fig. 3 shows the comparison on the performance of different schemes in terms of bit-error rate (BER). Clearly, the proposed scheme performs better than the existing codes when the SNR is larger than 13.5 dB.

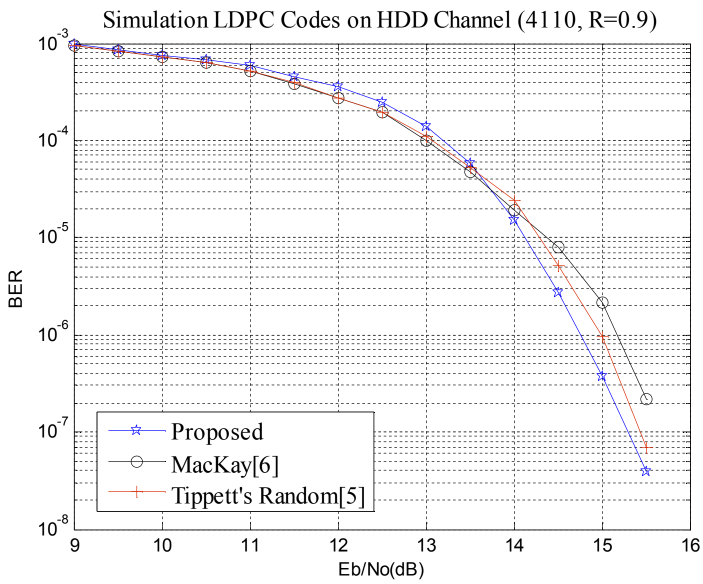

Fig. 3. Performance comparison.

This might be useful in hard disk drive (HDD) application, where a very low BER is required for a reliable HDD.

Conclusion

A new coding algorithm for LDPC codes was proposed using a combined method between the MSBA and TRN algorithms, where its performance was tested in a BPMR channel. At a block length of 4110 bit and a code rate of 0.9, the proposed code yields slightly better performance than the existing LDPC code. Because the proposed coding scheme has high complexity but slightly outperforms the existing codes, it can be concluded that the second task for encoding the parity bits does not substantially effect on the code performance.

References

[1] R. Gallager, Low-density Parity-check Code, IRE Transaction Information Theory, (1962), pp. 21-28.

[2] D. J. C. MacKay, Good error-correcting codes based on very sparse matrices, IEEE Transaction Information Theory 45 (2) (1999), pp. 399-431.

[3] M. Blaum, P. Farrell, and H. van Tilborg, “Array codes,” in Handbook of Coding Theory, V. S. Pless and W. C. Huffiman Eds., Elsevier 1998.

[4] C. Prasartkaew and S. Choomchuay, A design of parity check matrix for short irregular LDPC codes via magic square based algorithm, International Journal of electronics and communication engineering & technology (IJECET), 2013, pp. 146-160.

[5] C.Prasartkaew, P. Kovintavewat and S.Choomchuay, Parity-Check Matrix Construction of LDPC Codes in Bit-Patterned Media Recording Channels Using Tippett’s Random Number Table,Proceeding IEEE TENCON 2013.

[6] D. MacKay, Gallager code esources [Online], Available:http://www.inference.phy.cam.ac.uk/mackay/CodesFiles.html
[7] S. Nabavi, "Signal processing for bit-patterned media channel with inter-track interference," Ph.D. dissertation, Dept. Elect. Eng. Comp. Sci., Carnegie Mellon Univ., Pittsburgh, PA, 2008.
[8] B. Vucetic and J. Yuan, Turbo Codes: Principles and Applications, 2nd edition. MA: Kluwer, 2000.