Parity Check Matrix Construction of LDPC Codes in Bit-Patterned Media Recording Channels Using Tippett's Random Number Table

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***Abstract*—This paper presents the application of a random number table for the parity check matrix construction of irregular low-density parity check (LDPC) codes. The performance of the constructed codes was tested in bit-patterned media recording channels. The obtained performance of the proposed codes was compared with that of the existing codes. The simulation results show that, at a block length of 4110 bit and code rate of 0.9, the proposed LDPC code yields better performance than the existing ones, especially at high signal-to-noise ratio scenario.**

Keywords—Bit-patterned media recording; irregular LDPC code; random number table

I. INTRODUCTION

Current hard disk drives (HDDs) are based on perpendicular recording, which is approaching its storage limit a 1 terabits per square inch (Tb/in^2) [1]. To increase the storage capacity, new recording technologies must be employed for HDDs. Bit-patterned media recording (BPMR) is a promising candidate for future HDDs because it can achieve a recording density up to 4 Tb/in^2 and beyond [1].

In practice, a low-density parity check (LDPC) code is an outstanding error-correction code (ECC) because of its excellent performance close to Shannon's limit [2-3]. Generally, the performance of LDPC codes depends on their sparse parity check matrices [3].

Consequently, this paper proposes a novel construction algorithm of a parity check matrix for irregular LDPC codes, which can be used for arbitrary block length when it was designed with structured matrix and using a non-prime number parameter. Specifically, this parity check matrix is constructed using an application of referred Tippett's random number table [4] denoted as "TRN." The objective of this study is to design the parity check matrix with simple construction, simple encoding, good performance, and high code rate. Our designed matrix has a high code rate, which is suitable for BPMR channels. Simulation results indicate that the proposed LDPC code has less complexity and performs better than the previously proposed LDPC codes for BPMR channels.

II. LDPC CODES

LDPC codes are ECCs which became more popular and widely used for a wide area of applications, including communications and data storage systems. The main advantages of these codes are that they provide the performance close to the limited capacity for many different channels, and the decoding complexity is linear. Thus, the LDPC codes are suitable well for the parallel realization.

In general, LDPC codes are said to be *regular* if the number of "1" in each row or column is constant. If the parity check matrix $\boldsymbol{H}$ is low density but the number of "1" in each row or column is not constant, the code is said to be an *irregular* one.

A. Encoding

Similar to all other linear block codes, the parity check matrix $\boldsymbol{H}$ must satisfy the following relation:

$$\boldsymbol{C}_{(1\times n)}\boldsymbol{H}^T_{(n\times m)} = \boldsymbol{0} \tag{1}$$

where $\boldsymbol{C}$ is a codeword matrix, $\boldsymbol{H}$ is a parity check matrix, and a 1×m vector with all 0's. In a systematic form, $\boldsymbol{C}$ can be written as:

$$\boldsymbol{C}_{(1\times n)} = \left[\boldsymbol{p}_{(1\times n-m)} \,|\, \boldsymbol{m}_{(1\times m)}\right] \tag{2}$$

where $\boldsymbol{p}_{(1\times n-m)}$ represents the parity portion, and $\boldsymbol{m}_{(1\times m)}$ denotes the message portion. With $\boldsymbol{H} = [\boldsymbol{H}_1 \quad \boldsymbol{H}_2]$ or $\boldsymbol{H}^T = \begin{bmatrix}\boldsymbol{H}_1^T \\ \boldsymbol{H}_2^T\end{bmatrix}$ we can have

$$\boldsymbol{C}\boldsymbol{H}^T = [\boldsymbol{p}|\boldsymbol{m}]\begin{bmatrix}\boldsymbol{H}_1^T \\ \boldsymbol{H}_2^T\end{bmatrix} = \boldsymbol{m}\boldsymbol{H}_1^T + \boldsymbol{p}\boldsymbol{H}_2^T = \boldsymbol{0} \tag{3}$$

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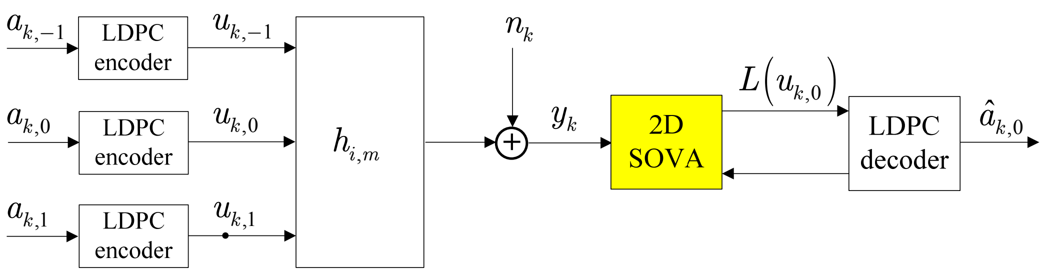

Fig. 1. A typical discrete-time BPMR channel model.

Then, the task of the encoder is to compute the parity matrix $\boldsymbol{p}$ that can be directly appended to the message to produce the codeword.

B. *Decoding*

There are several methods used to decode the LDPC codes, where each method was derived individually. These are, for instance, Believe Propagation (BP), Sum-Product (SP), and Message Passing (MP).

In the Log-domain Sum-Product algorithm [3], the message passes between the check nodes and the variable nodes. In each pass the log-likelihood ratio (*LLR*) is recorded, which is the probability of its likely symbol. In summary, the decoder goes through five steps as follows:

Step 1: Compute the initial value of $L_{(q_{ij})}$ transmitted from the variable node i to check node j; for all i; $1 \le i \le n$.

$$L_{(q_{ij})} = L_{(c_i)} = \frac{2y_i}{\sigma^2} = LLR_i = \log\left(\frac{p(c_i=0|y_i)}{p(c_i=1|y_i)}\right) \qquad (4)$$

where $L_{(c_i)}$ denotes log-likelihood ratio, σ^2 denotes the variance of additive white Gaussian noise (AWGN), and $p(c_i = 0|y_i)$ denotes the probability the $c_i = 0$ given that the input is y_i.

Step 2: Compute $L_{(r_{ji})}$ transmitted from the check node j to the variable node i, for all i and $1 \le i \le n$. Let $\phi(x) = log\left(\frac{e^x+1}{e^x-1}\right)$. Then, we obtain

$$L_{(r_{ji})} = \prod_{i'\epsilon V_{j/i}} \alpha_{i'j}\phi\left(\sum_{i'\epsilon V_{j/i}} \phi(\beta_{i'j})\right) \qquad (5)$$

where $\alpha_{ij} = sgn\{L(q_{ij})\}$ and $\beta_{ij} = |L(q_{ij})|$.

Step 3: Modify $L(q_{ij})$ and used it as the data transmitted from the variable node i to the check node j, for all i and $1 \le i \le n$.

$$L(q_{ij}) = L_{(c_i)} + \sum_{j'\epsilon V_{i/j}} L(r_{j'i}) \qquad (6)$$

Step 4: Compute the soft output according to

$$L(Q_i) = L_{(c_i)} + \sum_{j\epsilon C_i} L(r_{ji}) \qquad (7)$$

Step 5: The soft output obtained in step 4 is then employed in the hard decision as

$$\hat{c}_i = 1 \text{ if } L(Q_i) < 0 \text{, otherwise } \hat{c}_i = 0. \qquad (8)$$

III. BPMR CHANNEL

A typical discrete-time BPMR channel model is illustrated in Fig. 1, where we assume only two adjacent tracks cause most of the ITI. Therefore, the readback signal can be expressed as

$$y_k = \sum_i \sum_m h_{i,m} u_{k-i,m} + n_k \qquad (9)$$

where $u_{k,0}$, $u_{k,-1}$, and $u_{k,1} \in \{\pm 1\}$ represent an independent and identically distributed (*i.i.d.*) binary input bit sequences in the main track and the two adjacent tracks, respectively, $h_{i,m}$'s are the 2D channel response coefficients, and n_k is AWGN with zero mean and variance σ^2. Without the track mis-registration, we consider a discrete-time 3-by-3 symmetric channel response matrix of the form

$$H = \begin{bmatrix} h_{0,-1} & h_{1,-1} & h_{2,-1} \\ h_{0,0} & h_{1,0} & h_{2,0} \\ h_{0,1} & h_{1,1} & h_{2,1} \end{bmatrix} = \begin{bmatrix} 0.0347 & 0.2297 & 0.0347 \\ 0.1277 & 1 & 0.1277 \\ 0.0347 & 0.2297 & 0.0347 \end{bmatrix}, \qquad (10)$$

which is for a magnetic medium with SUL [1, 5]. Therefore, the sequence y_k is sent to a turbo equalizer, which iteratively exchanges soft information between the proposed two-dimensional (2D) SOVA equalizer and the LPDC decoder implemented based on the message passing algorithm with 3 internal iterations.

To perform maximum-likelihood (ML) equalization via the 2D SOVA, we propose to use a similar technique that was employed in the so-called bidirectional SOVA [6] to compute the LLR of the bit $u_{k,0}$, i.e., $L(u_{k,0})$. For this 3-by-3 channel matrix,

TABLE I. FIRST 40 NUMBERS FROM THE TIPPETT'S TABLE.

2952	6641	3992	9792	7979	5911	3170	5624
4167	9524	1545	1396	7203	5356	1300	2693
2370	7483	3408	2762	3563	1089	6913	7691
0506	5246	1112	6107	6008	8126	4433	8776
2754	9143	1405	9025	7002	6111	8816	6446

the trellis of this 2D SOVA will have 36 states. For each state at time k, there are 6 outgoing branches to 6 different states at time $k + 1$.

IV. RANDOM NUMBER TABLE

Random number tables have been used in statistics for tasks such as selected random samples. This was much more effective than manually selecting the random samples (with dice, cards, etc.). Tables of random numbers have the desired properties no matter how chosen from the table: by row, column, diagonal or irregularly. The first such table was published by L.H.C. Tippett in 1927 [7]. These numbers have been largely used with remarkable success not only in statistics but also for others fields, e.g., for conducting sampling experiments and simulations. This random number table was tested for its randomness by [8] through a variety of ways—the test results shown that these numbers can be considered to be random. The desired feature of these random numbers is that the entries are independent of each other.

Tippett's random number are by far the most popular ones and very much used. Table 1 gives the first 40 numbers from the Tippett's random number table.

A. Using random number tables

To select a random number from the random number, we could choose all required numbers at any point of table every two-digit number. It depends on where you start in the table, and whether we work down or across. Keep going until you have enough different numbers (two-digit) for filling in the sub-matrix as a shifting order. Fig. 2 illustrates the flowchart that describes how to use the random number table.

V. CONSTRUCTION OF PARITY CHECK MATRIX

The parity check matrix $\boldsymbol{H}$ can be constructed according to the following steps.

1. Define the designed parameters of J, K and L to be an integer greater than 3, where $\{J, K\} \leq L$. All values must satisfy the following condition:

$$(J \times K) - K - \lambda \leq L, \lambda = 5, 9, 14, \ldots \tag{11}$$

2. Place two-digit random number from Tippett's table in the structured parity check matrix $\boldsymbol{H}$ designed for irregular LDPC codes. These two-digit random numbers will be used as a shifting order of circulant matrix of $L \times L$.

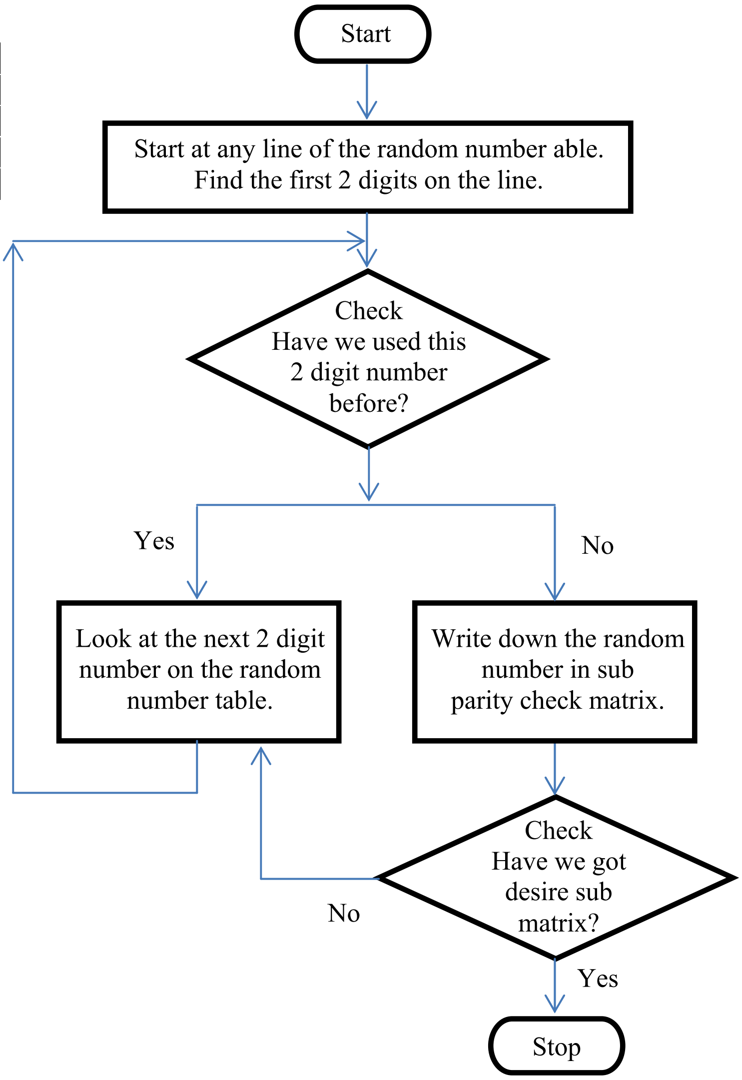

Fig. 2. Flowchart to use the random number table.

VI. SIMULATION RESULTS

The performance of the proposed codes (referred to as TRN) was investigated and compared with the existing comparable works, where their parity check matrices were generated based on Magic Square Theorem (denoted as MSBA) [9] and based on Random Gallager Codes (referred to as MacKay) [10]. The parameters used in our simulation are: $J = 3$, $K = 30$, and $L = 137$ such that all codes will have same block length of 4110 bits, the parity bits of $J \times L = 411$, the message bits of $K \times L = 3699$ and the code rate of $R = 1 - J/K = 0.9$. We also define

$$\mathrm{SNR} = 10\log_{10}\left(\frac{1}{R\sigma^2}\right), \tag{12}$$

in decibel (dB). Fig. 3 compares the performance of different LDPC codes after 30 iterations. Simulation results show that the proposed codes perform moderately better than the others.

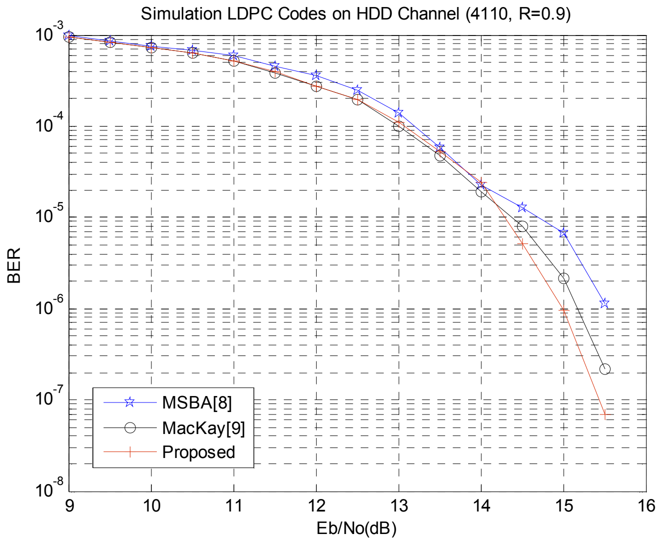

Fig. 3. BER performance comparison.

VII. Conclusion

The construction algorithm of LDPC codes using Tippett's random number table was proposed and the performance of the constructed codes was tested in BPMR channels. At a block length of 4110 bit and a code rate of 0.9, the proposed codes yield moderately better performance than the existing codes.

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