

Reduced-Complexity Modified Per-Survivor Iterative Timing Recovery Using M -Algorithm for Magnetic Recording System

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Abstract—A modified per-survivor iterative timing recovery (MPS-ITR), which jointly performs timing recovery, equalization, and error-correction decoding, has been proposed in [1] to deal with the problem of timing recovery operating at low signal-to-noise ratio. Practically, this scheme exploits a split-preamble strategy in conjunction with a per-survivor soft-output Viterbi algorithm equalizer to make it more robust against severe timing jitters or cycle slips. Although the MPS-ITR outperforms other iterative timing recovery schemes [1], it still has very high complexity. In this paper, we propose a reduced-complexity MPS-ITR scheme (denoted as MPS-ITR-M) to make it more implementable in real-life applications. This is achieved by applying the M -algorithm [2] to the MPS-ITR. Numerical results indicate that at low-to-moderate complexity, the MPS-ITR-M will perform better than other schemes.

Keywords—iterative timing recovery; per-survivor iterative timing recovery; M -algorithm; timing acquisition.

I. INTRODUCTION

Timing recovery is the process of synchronizing the sampler with the received analog signal. Sampling at the right times is critical to achieving good overall performance. The large coding gains of iterative error-correction codes (ECCs) enable reliable communication at very low signal-to-noise ratio (SNR). This means that timing recovery must be performed at an SNR lower than ever before. A conventional receiver performs timing recovery and turbo equalization [3] separately. Specifically, conventional timing recovery ignores the presence of ECCs and thus fails to work properly when the SNR is low enough.

To improve the performance of the conventional receiver, Kovintavewat *et al.* [4] proposed a per-survivor iterative timing recovery (PS-ITR) scheme, which jointly performs timing recovery, equalization, and error-correction decoding. It is realized by first applying the per-survivor processing (PSP) technique [5], a technique of jointly estimating a data sequence and unknown parameters, to the soft-output Viterbi algorithm (SOVA) [6], resulting in a per-survivor SOVA equalizer, denoted as “PSP-SOVA” [4]. Then, PSP-SOVA iteratively exchanges soft information with a soft-in soft-output (SISO) decoder. As investigated in [4], the PS-ITR outperforms the conventional receiver because it can automatically correct a cycle slip [7] after a few number of turbo iterations.

The M -algorithm was first introduced by Simmons and Mohan [2]. It has been employed in many applications, including source coding [2] and channel decoding [8]. Nevertheless, Iki *et al.* [9] proposed the application of M -algorithm and stack algorithm to the trellis shaping with peak-to-average power ratio reduction for single-carrier signal. Additionally, the M -algorithm has also been used in data storage systems. For instance, a simplified noise-predictive partial response maximum likelihood system in conjunction with the M -algorithm was proposed in [10] for dual-layered perpendicular magnetic recording channels

Because the MPS-ITR scheme has very high complexity, we therefore apply the M -algorithm [2] to the MPS-ITR, resulting in the reduced-complexity MPS-ITR scheme (denoted as MPS-ITR-M), so as to reduce its complexity and to make it more implementable in real-life applications. Additionally, we consider only a coded partial response channel in this paper because this channel is widely used in magnetic recording systems [11]. Thus, it will be shown later that at low-to-moderate complexity, the MPS-ITR-M performs better than the MPS-ITR and the PS-ITR.

This paper is organized as follows. After explaining the channel model in Section II, Section III briefly describes how the MPS-ITR-M works. Simulation results and discussion are given in Section IV. Finally, Section V concludes this paper.

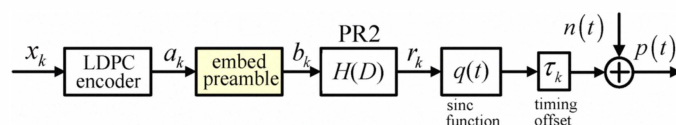


Figure 1. Data encoding with a PR2 channel model.

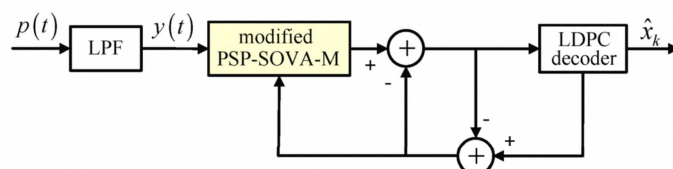


Figure 2. Proposed iterative timing recovery (i.e., MPS-ITR-M).

II. CHANNEL MODEL

Consider the coded partial-response (PR) channel in Fig. 1, where $H(D) = \sum_{k=0}^{\nu-1} h_k D^k = 1 + 2D + D^2$ is a PR2 channel, h_k is the k -th channel coefficient, D is the delay operator, and ν is channel memory. The message sequence $x_k \in \{0, 1\}$ is encoded by an error-correction encoded and is mapped to a binary sequence $a_k \in \{\pm 1\}$ of length L . Next, a preamble is inserted in a sequence a_k to obtain a sequence $b_k \in \{\pm 1\}$. The readback signal can then be written as

$$p(t) = \sum_k r_k q(t - kT - \tau_k) + n(t), \quad (1)$$

where $r_k = b_k * h_k \in \{0, \pm 2, \pm 4\}$ is the noiseless channel output, $*$ is the convolution operator, $q(t) = \sin(\pi t/T)/(\pi t/T)$ is an ideal zero-excess-bandwidth Nyquist pulse, T is a bit period, and $n(t)$ is an additive white Gaussian noise with a two-sided power spectral density $N_0/2$. The uncertainty in the timing is captured by the timing offset τ_k , which is modeled as a random walk [12] according to $\tau_{k+1} = \tau_k + \mathcal{N}(0, \sigma_w^2)$, where σ_w controls the severity of the timing jitter. The random walk model is chosen because of its simplicity and its ability to represent a variety of channels by changing only one parameter.

At the receiver, the readback signal $p(t)$ is filtered by an ideal low-pass filter (LPF), whose impulse response is $q(t)/T$, to eliminate the out-of-band noise, and is then sampled at time $kT + \hat{\tau}_k$, creating

$$y_k = y(kT + \hat{\tau}_k) = \sum_i r_i q(kT + \hat{\tau}_k - iT - \tau_i) + n_k, \quad (2)$$

where $\hat{\tau}_k$ is the receiver's estimate of τ_k , and n_k is *i.i.d.* zero-mean Gaussian random variable with variance $\sigma_n^2 = N_0 / (2T)$, i.e., $n_k \sim \mathcal{N}(0, \sigma_n^2)$.

Conventional timing recovery is based on a PLL [7], which consists of a timing error detector (TED), a loop filter, and a voltage-controlled oscillator (VCO). A decision-directed TED computes the receiver's estimate of the timing error $\varepsilon_k = \tau_k - \hat{\tau}_k$ using the well-known Mueller and Müller (M&M) TED algorithm [13] according to

$$\hat{\varepsilon}_k = \frac{6T}{40} \{y_k \hat{r}_{k-1} - y_{k-1} \hat{r}_k\}, \quad (3)$$

where \hat{r}_k is an estimate of r_k , and the constant $6T/40$ [14] is used to normalize the timing function of the M&M TED in (3) to have unit slope at origin [7]. For simplicity, we assume no frequency offset in the system. Thus, the next sampling phase offset can be updated by a first-order PLL according to

$$\hat{\tau}_{k+1} = \hat{\tau}_k + \alpha \hat{\varepsilon}_k, \quad (4)$$

where α is a PLL gain parameter [7].

III. PROPOSED SCHEME

The proposed scheme, i.e., MPS-ITR-M, will iteratively exchange soft information between a modified PSP-SOVA-M module and the SISO decoder as shown in Fig. 2. This modified PSP-SOVA-M is obtained by applying the M-algorithm [2] to the modified PSP-SOVA proposed in [1], which can be explained how it performs as follows.

The modified PSP-SOVA is developed based on the PSP-SOVA [4] with an aid of the new split-preamble strategy [1]. Specifically, the PSP-SOVA uses the conventional preamble arrangement, which places all C known bits at the beginning of the data sector. On the other hand, the modified PSP-SOVA uses a split a C -bit preamble into two parts. The first part of $C/2$ bits is placed at the beginning of the data sector, and the second part of $C/2$ bits is divided into $C/(2m)$ clusters (e.g., $m = 1, 2$, or 4 bits), each of which is then embedded uniformly within the user data stream. This split preamble is utilized to adjust the branch metric calculation in PSP-SOVA to ensure that the survivor path occurs in a correct direction. Based on extensive simulation search, we found that the modified PSP-SOVA with the one-bit split-preamble arrangement (i.e., $m = 1$) provides the best performance.

The M-algorithm [2] was originally proposed to reduce complexity of the Viterbi algorithm, which can be described how it works as follows. Fig. 3 shows the trellis diagram of the PR2 channel, which has 4 states. At each time instant, the M-algorithm first finds the minimum path metric leading to each trellis state. Hence, it retains only the M paths (M must be less than the total number of states in one stage of the trellis) with the lowest path metrics among all survivor paths. For example, in Fig. 3, if we assume that $M = 3$ and the state 0 has maximum path metric at time k . As a result, *only* three states (i.e., state 1, 2, and 3) will be used in branch metric calculation at the k -th stage.

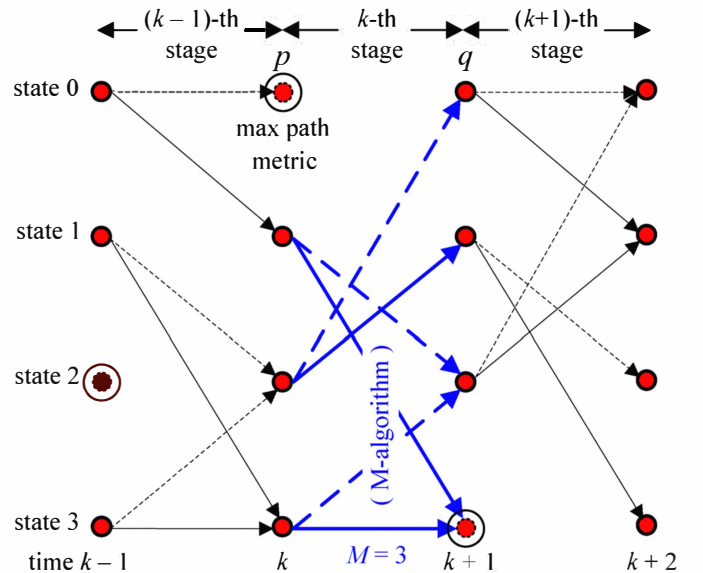


Figure 3. The PR2 trellis structure illustrating how M-algorithm performs.

Because the modified PSP-SOVA is developed based on the Viterbi algorithm, its complexity grows exponentially with channel memory. Therefore, to reduce its complexity, we apply the M-algorithm [2] to the modified PSP-SOVA, creating the modified PSP-SOVA-M. Fig. 4 shows the modified PSP-SOVA-M algorithm, where a constant $6T/40$ in (A-11) is only for the PR2 channel, which can be included in the PLL gain parameter. It should be noted that the modified PSP-SOVA-M works in a *same* manner as the modified PSP-SOVA [1] does, except that the modified PSP-SOVA-M has an *extra* step according to the M-algorithm, which can be briefly explained as follows.

Consider the trellis diagram in Fig. 3 at the k -th stage, where we denote \mathbf{M} as a set of all states (e.g., $\mathbf{M} = \{1, 2, 3\}$) that still remain at time k according to the M-algorithm. To reduce the number of states, a state p will be chosen from \mathbf{M} , i.e., $p \in \mathbf{M}$. In other words, only the branches emanating from \mathbf{M} will be used in branch metric calculation at the k -th stage.

IV. SIMULATION RESULTS

Consider a rate-8/9 system in which a block of 3640 message bits is encoded by a regular (3, 27) LDPC code [15], resulting in a coded block length of 4095 bits. The parity-check matrix has 3 ones in each column and 27 ones in each row. The SISO equalizer is implemented based on SOVA, whereas the SISO decoder is implemented based on the message-passing algorithm with 5 internal iterations ($N_{in} = 5$).

During an acquisition mode, the PLL gain parameters (α 's) for the conventional receiver and PS-ITR were designed to recover the phase change within 256 symbols (according to its preamble), whereas those for MPS-ITR, MPS-ITR-M1 ($M = 3$), and MPS-ITR-M2 ($M = 2$) were designed to recover the phase change within 128 symbols because the preamble was divided into two parts. Note that the α 's for all schemes were designed based on a linearized model of PLL [7], assuming that the S-curve slope is one at the origin, and there is no noise in the system. Furthermore, we consider the case where the α designed to recover the phase change within 256 symbols is used for all schemes during a tracking mode.

Fig. 5 compares the BER performance of different iterative timing recovery schemes at the 5-th iteration for the system with a severe random walk parameter $\sigma_w/T = 1.2\%$ (which implies a high probability of occurrence of cycle slips) as a function of per-bit SNRs, E_b/N_0 's. Apparently, the MPS-ITR-M1 performs better than the MPS-ITR-M2, the PS-ITR, and the conventional receiver. Furthermore, it is evident that for given the number of iterations, the MPS-ITR provides better performance than the other schemes because the MPS-ITR can reduce the occurrence of cycle slips and can also automatically correct a cycle slip much more efficiently than the PS-ITR [1]. Nevertheless, we will show that the MPS-ITR-M1 scheme can perform better than the MPS-ITR scheme when operating at low-to-moderate complexity.

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(A-1) Initialize  $\Phi_0(p) = 0$  for  $\forall p$ 
(A-2) Initialize  $\hat{\tau}_0(p) = 0$  for  $\forall p$ 
(A-3) For  $k = 0, 1, \dots, L + \nu - 1$ 
(A-4)   For  $q = 0, 1, \dots, Q - 1$ 
(A-5)      $y_k(p) = y_k(kT + \hat{\tau}_k(p))$  for  $p \in \mathbf{M}$ 
(A-6)      $\rho_k(p, q) = |y_k(p) - \hat{r}(p, q)|^2$  for  $p \in \mathbf{M}$ 
(A-7)     If  $k = i$  then [ $i$  is the preamble position]
           For  $p \in \mathbf{M}$ 
             If  $\hat{b}(p, q) \neq f_i$  then
                $\rho_k(p, q) = \Delta$  [ $\Delta$  is a large number]
           End
(A-8)      $\pi_{k+1}(q) = \arg \min_{p \in \mathbf{M}} \{\Phi_k(p) + \rho_k(p, q)\}$ 
(A-9)      $\Phi_{k+1}(q) = \Phi_k(\pi_{k+1}(q)) + \rho_k(\pi_{k+1}(q), q)$ 
(A-10)     $S_{k+1}(q) = [S_k(\pi_{k+1}(q)) | \pi_{k+1}(q)]$ 
(A-11)     $\hat{\epsilon} = \frac{6T}{40} \{y_k(\pi_{k+1}(q)) \hat{r}(\pi_k(\pi_{k+1}(q)), \pi_{k+1}(q))$ 
            $- y_{k-1}(\pi_k(\pi_{k+1}(q))) \hat{r}(\pi_k(q), q)\}$ 
(A-12)     $\hat{\tau}_{k+1}(q) = \hat{\tau}_k(\pi_{k+1}(q)) + \alpha \hat{\epsilon}$ 
(A-13)     $\Delta_{k+1}(q) = \max_{p \in \mathbf{M}} \{\Phi_k(p) + \rho_k(p, q)\} - \Phi_{k+1}(q)$ 
(A-14)    Initialize  $\hat{L}_k(q) = +\infty$  [Soft decision update]
(A-15)    For  $j = k - \nu, \dots, k - \delta$ 
           Compare the two paths merging
           in state  $q$  (i.e.,  $\Psi_{k+1} = q$ )
           If  $\hat{a}_j^{(1)}(\Psi_{j+1}) \neq \hat{a}_j^{(2)}(\Psi_{j+1})$  then
             Update  $\hat{L}_j(\Psi_{j+1}) = \min(\hat{L}_j(\Psi_{j+1}), \Delta_{k+1}(q))$ 
           End
(A-16)    If  $k \geq \delta$  then
           Output the soft decision according to  $\lambda_k^p = \hat{a}_{k-\delta} \hat{L}_{k-\delta}$ ,
           which can be extracted from the survivor path
           that minimizes  $\Phi_{k+1}$ 
           End
End
    
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Figure 4. The modified PSP-SOVA-M algorithm.

Fig. 6 compares the BER performance of different iterative timing recovery schemes when they have same complexity at $\sigma_w/T = 1.2\%$. It is apparent that the MPS-ITR-M1 performs better than other schemes. Consequently, it is worth employing the MPS-ITR-M1 in the system when the complexity is limited to a low-to-moderate amount.

V. CONCLUSIONS

We proposed a reduced-complexity modified per-survivor iterative timing recovery scheme to jointly perform timing recovery, equalization, and error-correction decoding. This scheme is obtained by applying the M-algorithm to the modified PSP-SOVA to make it more implementable in real-life applications. In addition, we found that the choice of M 's is crucial to the overall system performance. Specifically, the

M parameter mainly depends on the channel used. For the PR2 channel, $M = 3$ is a good choice for our proposed scheme. Simulation results show that at low-to-moderate complexity, the reduced-complexity modified per-survivor iterative timing recovery scheme (with $M = 3$) performs better than other iterative timing recovery schemes and the conventional receiver.

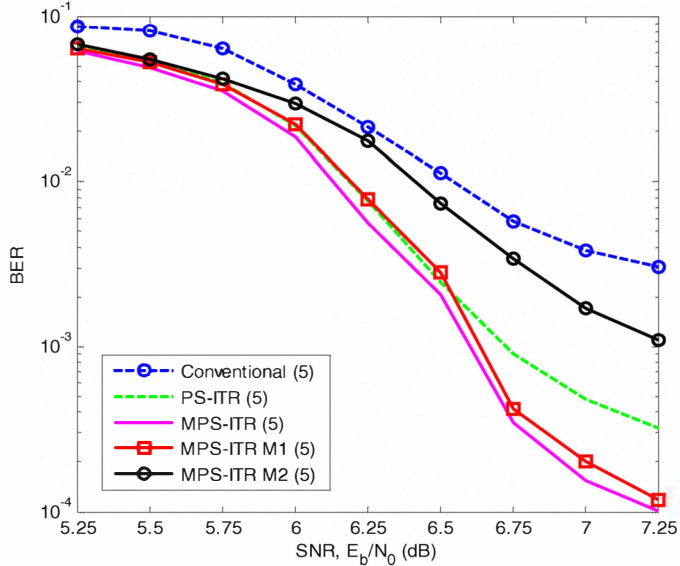


Figure 5. Performance comparison at the 5-th iteration when $\sigma_w/T = 1.2\%$.

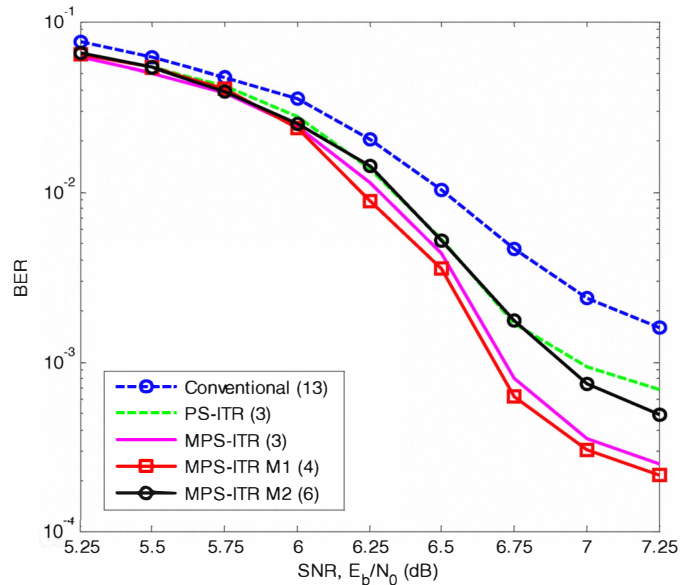


Figure 6. Performance comparison with same complexity at $\sigma_w/T = 1.2\%$.

ACKNOWLEDGMENT

This work was supported by National Electronics and Computer Technology Centre (NECTEC), National Science and Technology Development Agency (NSTDA) and IU CRC in Data Storage Technology and Applications (D⁺STAR), King Mongkut's Institute of Technology Ladkrabang under grant HDDA 50-001D.

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