A Construction of Parity-Check Matrix for Irregular LDPC Codes in Bit-Patterned Media Recording Channels

Chutima Prasartkaew[1], Piya Kovintavewat[2], and Somsak Choomchuay[3]
[1]*Department of Mathematics and Computer Technology, Faculty of Science and Technology, Rajamangaly University of Technology Thunyaburi, Thailand*
[2]*Data Storage Technology Research Center, Nakhon Pathom Rajabhat University, Thailand*
[3]*Faculty of Engineering, King Mongkut's Institute of Technology Ladkrabang, Thailand*
[1]*chutima@rmutt.ac.th*, [2]*piya@npru.ac.th*, [3]*kchsomsa@kmitl.ac.th*

Abstract

This paper presents the construction of parity check matrix for irregular low-density parity-check (LDPC) codes. We propose a new algorithm based on a magic square theorem to construction the parity check matrix. The performance of the constructed codes will be tested in bit-patterned media recording (BPMR) channels. At a block length of 4080 bit and code rate of 0.9, the simulation results show that the proposed LDPC code yields better performance than the existing ones, especially at high signal-to-noise ratio scenario.

Keywords: Bit-patterned media recording, irregular LDPC code, magic square theorem.

1. Introduction

BPMR is a promising candidate for future hard disk drives because it can achieve high recording density up to 1 Tb/in^2 and beyond. A low-density parity-check (LDPC) code is an outstanding error-correction code (ECC) because of its excellent performance close to Shannon's limit [1-2]. In general, the performance of LDPC codes depends on their sparse parity-check matrices.

Consequently, this paper proposes a novel parity-check matrix for irregular LDPC codes, which can be used for arbitrary block length when it was designed with structured matrix and using a non-prime number parameter. Specifically, this parity-check matrix is constructed using a novel algorithm based on a Magic Square Theorem [3], denoted as "MSA." The objective of this study is to design the parity-check matrix with simple construction, simple encoding, good performance, and high code rate. Our designed matrix has a high code rate, which is suitable for BPMR channels. Results show that the proposed LDPC code has less complexity and performs better than previously proposed LDPC codes in BPMR channels.

2. BPMR Channel

A typical discrete-time BPMR channel model is illustrated in Fig. 1, where we assume only two adjacent tracks cause most of the ITI. Thus, the readback signal can be expressed as

$$y_k = \sum_i \sum_m h_{i,m} u_{k-i,m} + n_k$$

where $u_{k,0}$, $u_{k,-1}$, and $u_{k,1}$ represent the random, uncoded binary (i.e., ± 1) input bit sequences in the main track and the two adjacent tracks, respectively, $h_{i,m}$'s are the 2D channel response coefficients, and n_k is AWGN with zero mean and variance σ^2. Without the track mis-registration, we consider a discrete-time 3-by-3 symmetric channel response matrix of the form $\mathbf{H}$ = [$h_{0,-1}$ $h_{1,-1}$ $h_{2,-1}$; $h_{0,0}$ $h_{1,0}$ $h_{2,0}$; $h_{0,1}$ $h_{1,1}$ $h_{2,1}$] = [0.0347 0.2297 0.0347; 0.1277 1 0.1277; 0.0347 0.2297 0.0347], which is for the media with SUL [4]. Hence, the sequence y_k is sent to a turbo equalizer, which iteratively exchanges soft information between the proposed two-dimensional (2D) SOVA equalizer and the LPDC decoder implemented based on the message passing algorithm with 3 internal iterations.

To perform maximum-likelihood (ML) equalization via a 2D SOVA, we propose to use a similar technique that was employed in the so-called bidirectional SOVA [5] to compute the log likelihood ratio (LLR) of the bit $u_{k,0}$, i.e., $L(u_{k,0})$. For this 3-by-3 channel matrix, the trellis of this 2D SOVA will have 36 states. For each state at time k, there are 6 outgoing branches to 6 different states at time k + 1.

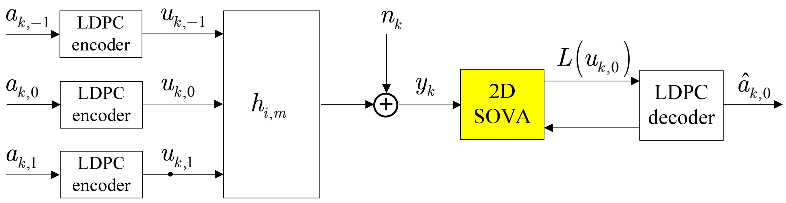

Fig. 1. A typical discrete-time BPMR channel model.

3. Construction of parity-check matrix

The parity-check matrix **P** can be constructed according to the following steps.

1) Define the designed parameters of J, K and L to be an integer greater than 3, where $\{J, K\} \leq L$. Only L value is used for sizing the magic square array, which will be generated as the next step. Moreover, all values must satisfy the following condition:

$$(J \times K) - K - \lambda \leq L, \quad \lambda = 5, 9, 14, \ldots$$

2) A magic square array of size $\geq L$ can be generated as
   - If L is odd number, the magic square array will be generated by using the Simon de la Loubère's algorithm.
   - If L is even number, the magic square array will be generated by using the Heinrich Cornelius Agrippa's algorithm.
3) Place this magic square array in the structured parity-check matrix **P** designed for irregular LDPC codes. Each number in the magic square array will be used as a shifting order of circulant matrix of $L \times L$.

4. Simulation results

In this study, the performance of the constructed code (referred to as MSA) is compared with the previous works, where their parity-check matrices was generated based on Chinese Remainder Theorem (denoted as Non-prime CRT) [6] and based on Size Compatible-Array (referred to as SC-Array) [7]. The parameters used in our simulation are: J = 3, K = 30, and L = 136 such that all codes will have same block length of 4080 bits, the parity bits of $J \times L$ = 408, the message bits of $K \times L$ = 3672 and the code rate of $R = 1 - J/K = 0.9$. We also define SNR = $10\log_{10}(1/(R\sigma^2))$ in decibel (dB). Fig. 2 compares the performance of different LDPC codes after 5 iterations. Clearly, the proposed LDPC code performs better than the others, especially at high SNR.

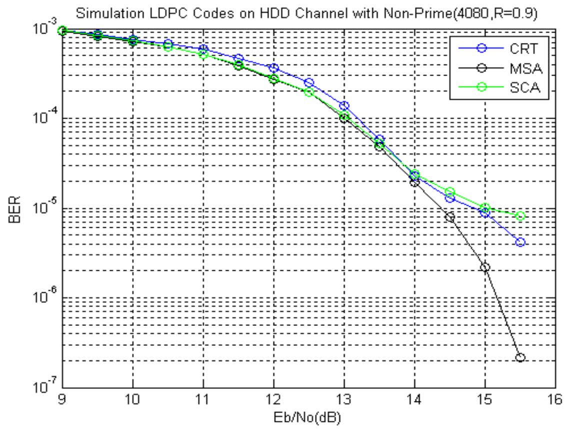

Fig. 2. BER performance comparison.

5. Conclusion

A new algorithm based on a magic square theorem was proposed for the parity-check matrix construction of the irregular LDPC codes. The performance of the constructed codes was tested in BPMR channels. At a block length of 4080 bit and a code rate of 0.9, the proposed codes yield better performance compared to the existing codes, especially at high SNRs.

References

[1] D. J. C. MacKay and R. M. Neal, "Near Shannon limit performance of low-density parity-check codes," *Elec. Lett.*, vol.32, pp.1645-1646, Aug. 1996.

[2] R. Gallager, "Low-density parity-check codes," *IRE Trans. Inform. Theory*, vol. IT-8, pp. 21-28, Jan. 1962.

[3] Y. H. Ku and N.-X. Chen, "Some theorems on construction of magic squares," *Journal of the Franklin Institute*, vol. 322, pp. 253-266, Nov-Dec 1986.

[4] S. Karakulak, "From channel modeling to signal processing for bit-patterned media recording," *Ph.D. dissertation, Department of Electrical Engineering, University of California*, San Diego, 2010.

[5] B. Vucetic and J. Yuan, *Turbo Codes: Principles and Applications*, 2nd edition. MA: Kluwer, 2000.

[6] C. Chusin *et al.*, "A Design of Nonprime Block Irregular LDPC Codes via CRT", *ISCIT Int. Symp. on Comm. and Inform. Tech.*, Sep. 28-30, 2010.

[7] D. Abematsu *et al.*, "Size Compatible (SC)-Array LDPC Codes" *Vehicular Technology Conference*, pp.1147-1151, 2007.