A Study of 2D Coding Schemes for ISI and ITI Mitigation in Bit-Patterned Media Recording Channels

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Abstract

The two-dimensional (2D) interference consisting of inter-symbol interference (ISI) and inter-track interference (ITI) is a crucial problem in bit patterned media recording (BPMR) systems. Because the ITI effect is more severe than the ISI effect, most of modulation codes focus merely on ITI condition. However, when an areal density (AD) is sufficiently high, the ISI effect begins to extremely affect the overall system performance, which cannot be ignored. Therefore, this paper studies the difference between the coding schemes based only on ITI and that based on ITI and ISI. With a similar methodology, we propose the rate-7/9 modulation code to deal solely with the ITI effect and the rate-6/9 modulation code to combat the 2D interference. Specifically, the proposed codes change an input data sequence before recording based on a look-up table so as to avoid the data patterns that easily cause an error at the data recovery process. Numerical results show that at the AD of 3 Tb/in^2, the rate-6/9 modulation code, which takes the ISI effect into consideration, is superior to the rate-7/9 one by 5 dB gain at bit-error rate of 10^{-4}.

Keywords: BPMR, 2D interference, Modulation code, 2D coding scheme.

1. Introduction

To achieve high recording densities in bit-patterned media recording (BPMR), the spacing between data bit islands in both the along-track and the across-track directions must be reduced, thus increasing the two-dimensional (2D) interference. In general, the 2D interference can deteriorate the overall system performance considerably.

Many 2D coding schemes [1-2] have recently been proposed to cope with the 2D interference. However, these 2D coding schemes were designed based on the channel model with overshoot, and only investigated the worst case scenario of the recorded bit pattern. For example, the 2D coding scheme with a code rate of 7/9 was specially designed in [1] to avoid the destructive case, where the redundant bits were placed in fixed positions at every 3-by-3 data array. Nonetheless, this coding scheme had a drawback that the redundant bits had no error correction ability. Then, Shao *et al.* [2] introduced another 2D coding scheme with a code rate of 5/6, which had lower redundancy and yielded better performance than the code presented in [1]. Moreover, a multi-track recorded-bit patterning (M-RBP) scheme was proposed in [3] to reduce the effect of 2D interference based on circular shift and the weight calculation methods. Although this code had no redundant bits (i.e., a code rate of 1), it had high complexity and required a large buffer memory. As a consequence, the rate-4/5 modulation coding scheme [4] was introduced to combat only the ITI effect based on a look-up table, which has low complexity. Even though this coding scheme can improve the system performance in terms of bit-error rate (BER), it does not take the ISI effect into account during the code design

Therefore, we study the 2D coding scheme to alleviate both ISI and ITI in this paper. Based on a similar methodology, we propose the rate-7/9 coding scheme to alleviate only the destructive ITI and the rate-6/9 coding scheme to subside both the destructive ITI and ISI. Specifically, the proposed codes rearrange an input data sequence before recording it onto a magnetic medium based on a look-up table so as to avoid the data patterns that easily cause an error at the data detection process. It will be shown in simulation that the rate-6/9 coding scheme (designed based on both the ISI and ITI effects) can perform better than the rate-7/9 one (designed based only on the ITI effect).

The rest of this paper is organized as follows. After describing a BPMR channel model in Section 2, Section 3

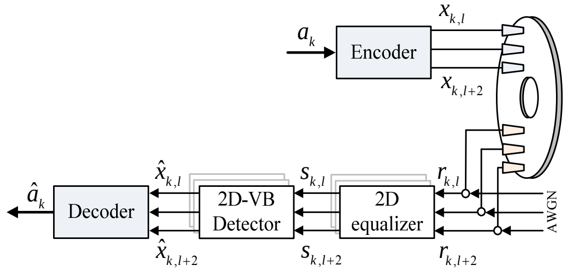

Fig. 1. A BPMR channel model with the proposed coding schemes.

explains how the proposed coding schemes perform. Simulation results are given in Section 4. Finally, Section 5 concludes this paper.

2. BPMR Channel Model

Consider a multi-track multi-head BPMR system [3-7] with the proposed coding schemes in Fig. 1. A binary input sequence $a_k \in \{\pm 1\}$ with bit period T_x is encoded by the proposed encoder to obtain three data tracks $\{x_{k,l}, x_{k,l+1}, x_{k,l+2}\}$ before recording them onto a medium. The readback signal from the k^{th} data bit on the l^{th} track can be written as [3-5]

$$r_{k,l} = x_{k,l} \otimes h_{k,l} + w_{k,l} = \sum_n \sum_m h_{m,n} x_{k-m,l-n} + w_{k,l}, \quad (1)$$

where $x_{k,l}$'s are the recorded bits, $\otimes$ is a 2D convolution operator, $h_{m,n}$'s are the 2D channel response coefficients [6, 8], m and n are the time indices of the bit island in the along-track and the across-track directions, and $w_{k,l}$ is an additive white Gaussian noise (AWGN) with zero mean and variance σ^2.

Practically, the 2D channel response coefficients are given by $h_{m,n} = P(mT_x, nT_z)$ [6], where $P(x, z)$ is a 2D Gaussian pulse response, x and z are the time indices in the along-track and the across-track directions, $\{m, n\} \in (-L, ..., 0, ..., L)$, $2L+1$ is the length of $P(x, z)$, and L is an integer. For simplicity, we use $L = 1$ to guarantee that the tail amplitude of $P(x, z)$ is small. Additionally, this paper considers the 2D Gaussian pulse response of the form [8]

$$P(x,z) = A\exp\left\{-\frac{1}{2c^2}\left[\left(\frac{x+\Delta_x}{\mathrm{PW}_x}\right)^2 + \left(\frac{z+\Delta_z}{\mathrm{PW}_z}\right)^2\right]\right\}, \quad (2)$$

where $A = 1$ is assumed to be the peak amplitude of the pulse response, $c = 1/2.3548$ is a constant to account for the relationship between PW_{50} and the standard deviation of a Gaussian pulse [8], Δ_x is the along-track location fluctuation (or position jitter [6]), Δ_z is the across-track location fluctuation, PW_x is the PW_{50} of the along-track pulse, and PW_z is the PW_{50} of the across-track pulse. Here, we assume that the position jitters (Δ_x and Δ_z) are modeled as a truncated Gaussian probability distribution function with zero mean and σ_j^2, where σ_j is specified as the percentage of T_x.

At the receiver, the readback data sequence $\{r_{k,l}\}$ is equalized by a 2D equalizer to obtained a sequence $\{s_{k,l}\}$, and is then sent to the 2D Viterbi detector [6, 7] to determine the most likely recorded sequence $\{\hat{x}_{k,l}\}$. Finally, the proposed decoder is employed to decode the 3-track data sequence $\{\hat{x}_{k,l}\}$ into a single track data sequence of an estimated input data sequence $\hat{a}_k$.

3. Proposed Coding Schemes

In general, the effect of 2D interference experienced in the detected bit can be either constructive or destructive, depending on the readback waveform of the detected bit and its surrounding bits [2]. Specifically, given the signal amplitude of an isolated bit (or one single bit), the constructive/destructive interference will amplify/decrease its signal amplitude. Hence, when the readback signal of the detected bit encounters the destructive interference, it could easily cause an error at the data detection process.

This paper proposes two coding schemes, whose code rate are 6/9 and 7/9, to alleviate the effect of 2D interference. Specifically, each of $u \in \{6, 7\}$ user bits is mapped to a 9-bit codeword, $\tilde{\mathbf{x}}$, in the form of 3-by-3 data array, i.e., 3 bits from the l^{th} track [$\tilde{x}_{k,l}$, $\tilde{x}_{k+1,l}$, $\tilde{x}_{k+2,l}$], 3 bits from the $(l + 1)^{\text{th}}$ track [$\tilde{x}_{k,l+1}$, $\tilde{x}_{k+1,l+1}$, $\tilde{x}_{k+2,l+1}$], and 3 bits from the $(l + 2)^{\text{th}}$ track [$\tilde{x}_{k,l+2}$, $\tilde{x}_{k+1,l+2}$, $\tilde{x}_{k+2,l+2}$], as shown in Fig. 2. Below are the details on how to design the proposed coding schemes.

By analyzing all destructive data patterns in the M-RBP scheme [3], we found that all of them occur when the k^{th} bit of the adjacent tracks differ from that of the center track, i.e., $[a_{k,l-1}, a_{k,l}, a_{k,l+1}]^T = [+1\ {-1}\ {+1}]^T$ or $[-1\ {+1}\ {-1}]^T$. Hence, we exploit this result to design our coding scheme to prevent such destructive data patterns to be written onto a medium. With 3-by-1 data array, there are 6 data patterns without $[+1\ {-1}\ {+1}]^T$ and $[-1\ {+1}\ {-1}]^T$. Then, we concatenate three 3-by-1 data arrays to obtain $6^3 = 216$ 3-by-3 data patterns that can be employed for our codewords.

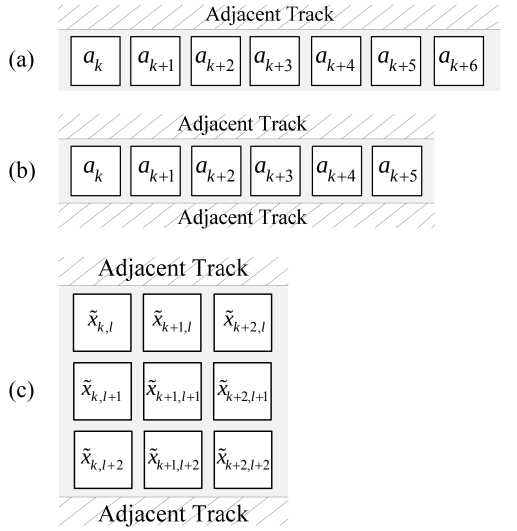

Fig. 2. Proposed coding schemes for a 3-track channel model, where (a) 7 user bits for a rate-7/9 modulation code and (b) 6 user bits for a rate-6/9 one are converted to (c) a codeword of 9 recorded bits.

To obtain 2^7 = 128 patterns for the proposed rate-7/9 modulation code, we select the first 128 patterns with the highest weight [5] from these 216 3-by-3 data patterns. Note that the data pattern with the maximum weight in [5] will cause the least destructive effect of 2D interference in the readback signal. Hence, each 3-by-3 codeword is mapped to each 7 user bits as depicted in Fig. 2 (a) and (c).

Moreover, we also found that out of 216 3-by-3 data patterns there are 102 patterns that have no [+1 –1 +1] and [–1 +1 –1] in both the along-track and the across-track directions. It should be pointed out that the data pattern [+1 –1 +1] or [–1 +1 –1] in the across-track direction will yield severe ITI effect, whereas that in the along-track direction will cause severe ISI effect. Consequently, to obtain a rate-6/9 modulation code, we need 2^6 = 64 3-by-3 data patterns to represent each of 6 user bits as illustrated in Fig. 2 (b) and (c). To achieve this, we again choose the first 64 patterns with the highest weight [5] from these 102 3-by-3 data patterns.

Therefore, we propose two modulation codes for a multi-track multi-head BPMR system, where the encoding and decoding processes are performed based on a look-up table. Notice that the rate-7/9 modulation code is designed to reduce the ITI effect only, while the rate-6/9 modulation code is designed to mitigate both the ISI and the ITI effects. In addition, the proposed coding schemes avoid all fatal data patterns that easily yield an error at the data recovery process to be recorded onto a medium. Finally, the recorded data sequence $\{x_{k,l}\}$ is the codeword $\tilde{\mathbf{x}}$ obtained from the proposed modulation encoder.

To decode an estimated 3-track recorded sequence $\{\hat{x}_{k,l}\}$, we use the same look-up table obtained from the encoding process to determine an estimated input data sequence $\{\hat{a}_k\}$. Because of AWGN, the decoding process might unable to perform correctly for some data patterns $\hat{\mathbf{x}}$ that are inconsistent with the codewords, i.e., $\hat{\mathbf{x}} \neq \tilde{\mathbf{x}}$. To solve this problem, we apply an Euclidean distance concept to measure the resemblance between $\hat{\mathbf{x}}$ and $\tilde{\mathbf{x}}$ in the decoder [5]. Specifically, for each codeword $\tilde{\mathbf{x}}$, the proposed decoder computes the Euclidean distance of $\hat{\mathbf{x}}$ according to

$$d\left(\hat{\mathbf{x}},\tilde{\mathbf{x}}\right)=\sqrt{\sum_{m=0}^{2}\sum_{n=0}^{2}\left(\hat{x}_{k+m,l+n}-\tilde{x}_{k+m,l+n}\right)^{2}} \tag{3}$$

Then, the estimated input data vector $\hat{\mathbf{a}}_k$ = [$\hat{a}_k$, $\hat{a}_{k+1}$, $\hat{a}_{k+2}$, $\hat{a}_{k+3}$, $\hat{a}_{k+4}$, $\hat{a}_{k+5}$, $\hat{a}_{k+6}$] for a rate-7/9 coding scheme or $\hat{\mathbf{a}}_k$ = [$\hat{a}_k$, $\hat{a}_{k+1}$, $\hat{a}_{k+2}$, $\hat{a}_{k+3}$, $\hat{a}_{k+4}$, $\hat{a}_{k+5}$] for a rate-6/9 coding scheme, corresponding to the codeword $\tilde{\mathbf{x}}$ that yields the minimum $d\left(\hat{\mathbf{x}},\tilde{\mathbf{x}}\right)$ in (3) will be the decoder output.

4. Simulation Results

We test the proposed coding schemes in the BPMR system illustrated in Fig. 1, where the two outer tracks (l – 1) and (l + 3) contain random data. For the conventional system (no coding), an input sequence a_k is written into a single track with random data on adjacent tracks. The signal-to-noise ratio (SNR) in dB is defined as SNR = $10\log_{10}(1/R\sigma^2)$, where R is a code rate (7/9 or 6/9) and σ is a standard deviation of AWGN. The 2D 3-by-3 target and 2D 3-by-7 equalizer are designed based on a minimum mean-squared error approach [6, 7] at the SNR required to achieve bit-error rate (BER) of 10^{-4}. Furthermore, the 2D Viterbi detector for this 3-by-3 symmetric target employs the trellis having 36 states with 6 parallel branches between any two connected states [7]. In simulation, each BER is computed based on a minimum number of 500 erroneous bits, and one data sector consists of 4095 bits.

We consider the case where $T_x = T_z$, and investigate the performance of the proposed coding schemes at high areal densities (ADs) of 2.5 and 3.0 Tb/in^2, which correspond to T_x equal to 16 nm and 14.5 nm, respectively. The 2D Gaussian pulse has the along-track PW$_{50}$ of 19.4 nm and the across-track PW$_{50}$ of 24.8 nm, similar to that considered in [6].

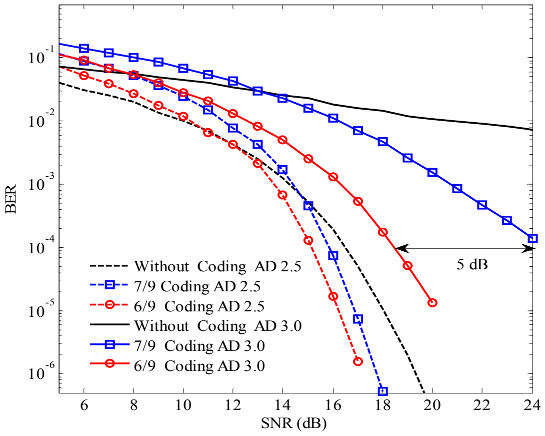

Fig. 3. BER performance at different areal densities without position jitter.

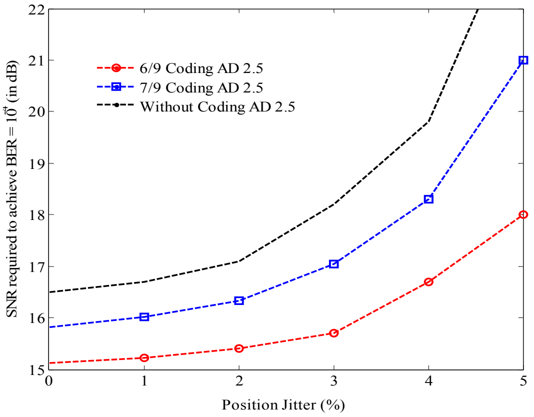

Fig. 4. Performance comparison for various position jitter amounts at AD = 2.5 Tb/in^2.

Fig. 3 compare the BER performance of the system with and without proposed coding schemes at the ADs of 2.5 and 3 Tb/in^2 without position jitter (i.e., σ_j = 0%). At AD = 2.5 Tb/in^2, it is evident that the system with the rate-6/9 coding scheme performs better than the rate-7/9 one and the system without coding for about 0.8 dB and 1.5 dB at BER = 10^{-4}, respectively. However, AD is increased to 3.0 Tb/in^2, the rate-6/9 coding scheme yields around 5 dB over the rate-7/9 one at BER = 10^{-4} and is far superior to the system without coding.

We also compare the performance of different coding schemes by plotting the SNR required to achieve BER = 10^{-4} as a function of position jitter amounts at AD = 2.5 Tb/in^2 in Fig. 4. Clearly, the system with the rate-6/9 coding scheme is superior to the system without coding, especially when the position jitter is large. Therefore, the proposed coding schemes are worth employing in the BPRM system, especially at high AD.

5. Conclusion

At high areal densities (more than 2.5 Tb/in^2), the 2D interference consists of ISI and ITI is very severe in the BPMR system. Therefore, this paper proposed the rate-7/9 and rate-6/9 coding schemes to alleviate not only the ITI effect but also the ISI effect. Simulation results indicate that the both coding schemes can improve the overall system performance, especially when the AD is high and the position jitter is large.

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