GEOMETRIC SEQUENCES

ลำดับเรขาคณิต

A geometric sequence is a sequence of numbers where each term after the first term is found by multiplying the previous one by a fixed non-zero number called the common ratio.

Common ratio : r

Determine which of the following sequences are geometric. If so, give the value of the common ratio, r

① 3, 6, 12, 24, 48, 96, ... Yes! $r = 2$ เพราะคูณตัวก่อนหน้าด้วยจำนวนเดิมไปเรื่อยๆ

② 3, $\frac{3}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{3}{16}$, $\frac{3}{32}$, $\frac{3}{64}$, ... Yes! $r = \frac{1}{2}$

③ 10, 15, 20, 25, 30, ... No! not geometric

④ $-1, 0.1, -0.01, 0.001, -0.0001, \ldots$

$-1, \frac{1}{10}, -\frac{1}{100}, \frac{1}{1000}, -\frac{1}{10000}, \ldots$ } Yes! $r = -\frac{1}{10}$ หรือ -0.1

GEOMETRIC SEQUENCES

Finding a formula for the "n-th" term. a_n

$$\boxed{a_n = a_1 \cdot r^{n-1}}$$

Consider the geometric sequence

3, 6, 12, 24, 48, ...

① Derive the a_n formula

② Find a_{10}

③ If $a_2 = 32$; $a_5 = -4$; find a_9 (~~earlier~~ Next video)

SOVING PROBLEMS INVOLVING ARITHMETIC SEQUENCES

$$a_n = a_1 + (n-1)d$$

b) Find the general term of an arithmetic sequence if $a_{21} = -14$ and $a_{51} = 226$

Sol^n

$$a_n = a_1 + (n-1)d$$

$$a_{21} = \boxed{a_1 + (21-1)d} = -14$$

$$a_{51} = \boxed{a_1 + (51-1)d} = 226$$

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$$-a_1 - 20d = 14$$

$$+a_1 + 50d = 226 \quad +$$

$$30d = 240$$

$$d = \frac{240}{30} = 8$$

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$a_1 + 20(8) = -14$

$a_1 + 160 = -14$

$a_1 = -174$

∴ General term ป.ต

$a_n = -174 + (n-1)8$

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Consider the geometric sequences

Ex) 3, 6, 12, 24, 48, ...

① Derive the formula (a_n)

$a_1 = 3$
$a_2 = 6$
$a_3 = 12$
$a_4 = 24$
$a_5 = 48$
⋮ ⋮

สังเกตได้ว่าพจน์ต่อเนื่องจะมีค่าเพิ่มขึ้นเป็น r เท่าของพจน์ก่อนหน้า

r = Common ratio

$a_1 = 3$
$a_2 = 3 \cdot 2^1$
$a_3 = 3 \cdot 2^2$
$a_4 = 3 \cdot 2^3$
$a_5 = 3 \cdot 2^4$
$a_6 = 3 \cdot 2^5$

$a_n = a_1 \cdot r^{?}$

$\boxed{a_n = a_1 \cdot r^{n-1}}$ ###

Ex $a_8 = ?$

$a_8 = 3(2)^{8-1} = 3(2)^7 = 3(128) = 384$ ###

SEQUENCES ลำดับเลขคณิต

\#

Ex1 1, 2, 3, 4, ... $\xrightarrow{\text{ค่าใกล้}}$ ∞ (Diverges) ลู่ออก

Ex2 1, 0.1, 0.01, 0.001, 0.0001, ... $\xrightarrow{\text{ค่าใกล้}}$ 0 (Converges) ลู่เข้า

a_1 a_2 a_3

Ex $a_n = \dfrac{3(-1)^n}{n!}$

แฟกทอเรียล Ex $5! = 5\times4\times3\times2\times1$

$1, 2, 3, 4, \cdots$

a_1, a_2, a_3, a_4

$a_1 = \dfrac{3(-1)^1}{1!} = \dfrac{-3}{1} = -3$

$a_2 = \dfrac{3(-1)^2}{2!} = \dfrac{3}{2} = 3/2$

$a_3 = \dfrac{3(-1)^3}{3!} = \dfrac{-3}{6} = -1/2$

ดังนั้น $-3, \frac{3}{2}, \frac{-1}{2}, \cdots$ ——— \#\#\#

\# พิสูจน์ลำดับเลขคณิตที่ใดๆ Converges or Diverges ?

Ex) $a_n = n(n-1)$

$\lim_{n\to\infty} n(n-1) = \infty(\infty-1) = \infty \Rightarrow$ Diverges

Ex) $\lim_{n\to\infty} \dfrac{2^n}{3^{n+1}} \Rightarrow \dfrac{\infty}{\infty}$ หาค่าไม่ได้

ดังนั้นต้องหาว่า lim ที่มีรูปแบบ ∞/∞ ใดๆ

วิธีทำ $\lim_{n\to\infty} \dfrac{2^n}{3^{n+1}} = \lim_{n\to\infty} \dfrac{2^n}{3^1 \cdot 3^n} = \lim_{n\to\infty} \dfrac{1}{3}\dfrac{2^n}{3^n} = \dfrac{1}{3}\lim_{n\to\infty}\left(\dfrac{2}{3}\right)^n$

$= \dfrac{1}{3}\left(\dfrac{2}{3}\right)^\infty = \dfrac{1}{3}(0) = 0 \Rightarrow$ Converges

r = ratio (Converges if $-1 < r < 1$)

SEQUENCES ลำดับเลขคณิต

\# Converges or Diverges ?

Ex) $\left\{ \frac{(2n-1)!}{(2n+1)!} \right\}$

take limit ดู a^r

$$\lim_{n\to\infty} \frac{(2n-1)!}{(2n+1)!} = \lim_{n\to\infty} \frac{(2n-1)\cdot(2n-2)\cdot(2n-3)\cdots(3)(2)(1)}{(2n+1)\cdot(2n)\cdot(2n-1)(2n-2)\cdots(3)(2)(1)}$$

$$= \lim_{n\to\infty} \frac{1}{(2n+1)(2n)} = 0 \Rightarrow \text{Converges} \quad \#\#\#$$

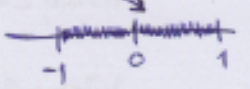
Ex) $a_n = \frac{\cos^2 n}{2^n}$

take limit ดู a^r

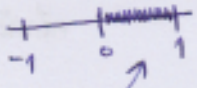
$$\lim_{n\to\infty} \frac{\cos^2 n}{2^n}$$

เพราะว่า $-1 \le \cos(n) \le 1$

$0 \le \cos^2(n) \le 1$



$$0 \le \frac{\cos^2(n)}{2^n} \le \frac{1}{2^n}$$



$\therefore \lim_{n\to\infty} 0 = 0$ และ $\lim_{n\to\infty} \frac{1}{2^n} = 0$

$\therefore \lim_{n\to\infty} \frac{\cos^2(n)}{2^n} = 0 \Rightarrow \text{Converges} \quad \#\#\#$

\# Squeeze Theorem \#

! FACTORIALS !

$5! = (5).(4).(3).(2)(1) = 120$ #

$6! = (6).(5).(4)(3).(2)(1) = 6(120) = 720$ #

$0! = 1$ #

Ex) $3! + 4! = ?$

Sol^n $(3).(2)(1) + (4).(3).(2) = 6 + 24 = 30$ #

Ex) $5! . 3! = ?$

Sol^n $(5)(4)(3).(2)(1).\ (3)(2).(1) = (120)(6) = 720$ #

Ex $\frac{8!}{3!} = \frac{8.7.6.5.4.3.2.1}{3.2.1} = \frac{40320}{6} = 6720$ #

Ex $\frac{100!}{98!} = \frac{(100)(99)(98)(97)(96)\ldots(3)(2)(1)}{(98)(97)(96)\ldots(3)(2)(1)} = (100)(99) = 9900$ #