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## A New Discrete Markov Chain Model of Binary Exponential Backoff Algorithm for Wireless Local Area Networks

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### Abstract

In this research, we propose a new discrete Markov chain model of binary exponential backoff Algorithm (BEB) for distributed coordination function (DCF) in IEEE802.11 wireless local area networks (WLANs). A new model uses Fixed Backoff stages and Fixed Contention windows (FBFC) technique on carrier sensing multiple accesses with collision avoidance and request-to-send clear-to-send protocol (CSMA/CA RTS CTS). The throughput efficiency of FBFC model is compared with the legacy discrete Markov chain model of BEB Algorithm. The legacy model is called Bianchi's model which is the original model for performance analysis of WLAN system. The FBFC model represents a new mathematical summation of the transmission probability parameter which can calculate the average saturation throughput of IEEE802.11a/b/g WLAN standards. The accuracy of transmission probability parameter is derived step by step under the global balance equation concept. The saturation throughputs of all models are compared under the same Physical layer (PHY) parameters and the same medium access control (MAC) scheme. Our numerical results show that the saturation throughput performance of FBFC technique is stable when the number of contending stations is increased in service area, or the WLAN system is in the high traffic load conditions. The distinction of FBFC scheme is low complexity and more realistic than the previous discrete Markov chain model.

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*Keywords:* Backoff Algorithm, Binary Exponential Backoff, Fixed Backoff stages and Fixed Contention windows, CSMA/CA RTS CTS, IEEE802.11a/b/g, Discrete Markov Chain Model

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### 1. Introduction

Today, the wireless local area network (WLAN) is becoming increasingly important, and the IEEE802.11 is one of the most popular standards in WLAN systems. Currently, the wireless local area networks solve the

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collision problems by using backoff algorithm scheme. High channel throughput and low packet delay are two important characteristics of a good backoff algorithm. A big problem of old backoff algorithm is that the throughput performances are unstable when the numbers of contending stations in service area is increased. The popular model for performance analysis of backoff algorithm is Bianchi's model in [4]. The author in [4] proposed two-dimension discrete Markov chain model of binary exponential backoff (BEB) algorithm under the finite number of contending stations. Afterward, the author in [5] proposed the discrete Markov chain model of binary negative-exponential backoff (BNEB) algorithm. The performance of BNEB algorithm is better than BEB algorithm when the number of contending stations is larger than 4 stations. However, the transmission probability ( $\tau$ ) in [4] and [5] are derived from the two-dimension discrete Markov chain model in general case (unlimited backoff stages and unlimited contention window sizes) so that the node's packet transmission probability ( $\tau$ ) in general case is high complexity and difficult understanding. The difference in this research is that the node's packet transmission probability of each backoff algorithm is derived by used the Fixed Backoff stages and Fixed Contention window sizes (FBFC) scheme. In FBFC technique, the accuracy of transmission probability is derived from step by step procedure by used the global balance equation concept in discrete Markov chain theory. The limitation in this research, we assume that:

- channel is ideal condition and no capture effect
- channel is divided into time slots of equal periods all contending stations
- channel is saturated condition, and the collision probability ( $p$ ) occurs when the WLAN channel has more than one contending stations to transmit a packet in a same timeslot
- the collision probability is constant and independent from the collision in the past
- all stations know the total of station in service area
- the transmission probability occurs when the contention window sizes are counted down to zero, and the transmission probability is unknown and to be solved
- data frame length is the same for all contending stations
- the contending stations ( $n$ ) are in the rang of each other
- the contending stations are fixed and known

The paper is organized as follows: In section II, we review discrete Markov chain model for IEEE802.11 WLAN. In section III, the fixed backoff stages and fixed contention windows technique is analyzed. In section IV, we introduce carrier sensing multiple accesses with collision avoidance protocol. The numerical results discuss in section V. Finally, section VI is the conclusion.

## 2. Discrete Markov Chain Model

### 2.1. The legacy of discrete Markov chain model or Bianchi's model

In 2000 year, Giuseppe Bianchi in [4] develops a simple discrete Markov chain model for the distributed coordination function mode in IEEE802.11 standards. Bianchi's model is shown in Fig.1. The transmission probability that depends on the collision probability ( $p$ ), the maximum backoff stages ( $m$ ), and the contention window (CW) sizes is given by

$$\tau_{[Bianchi's\ model]} = \frac{2(1-2p)(1-p)^m}{(1-2p) + CW[(1-2p) + p(1-(2p)^m)]} \quad (1)$$

In wireless local area network system, the packet collision occurs whenever more than one contending station or node tries to access the medium at the same timeslot. Backoff algorithm is a technique to solve the collision problems. In Bianchi's model, binary exponential backoff algorithm is used to reduce the data packet collision. At the first transmission of a contending node, if the channel is idle for more than a distributed coordination function inter-frame space (DIFS) time, a contending station can transmit immediately. If the channel is busy, the contending station will generate a random contention window sizes.

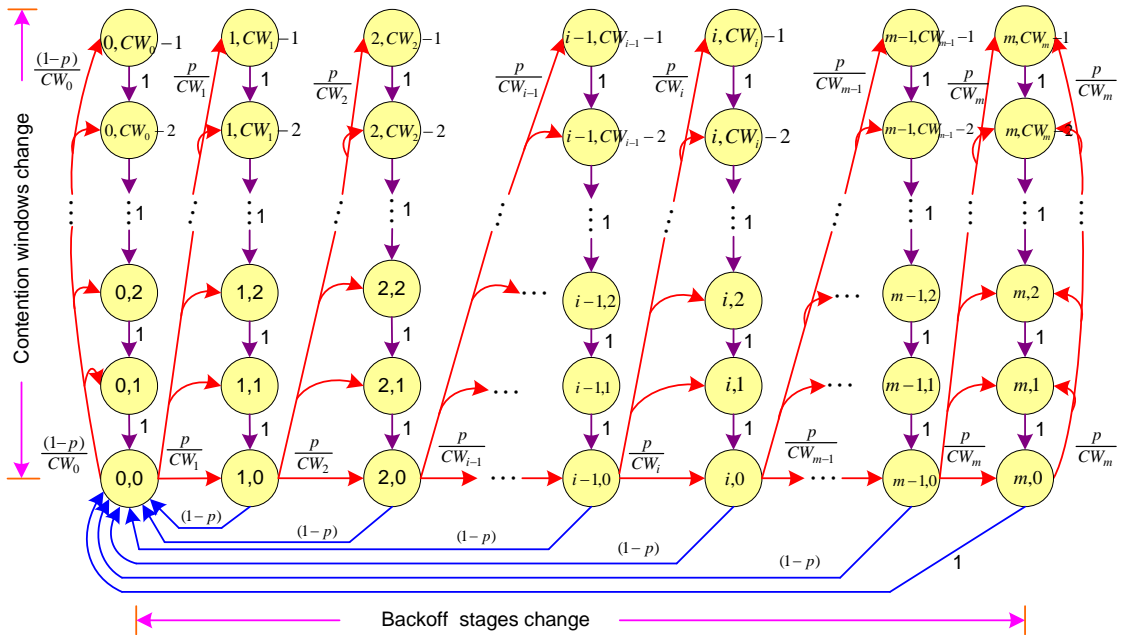


Fig. 1. Two dimension discrete Markov chain model in general form (Bianchi's model)

At the first transmission, the contention window size is selected equal to a minimum contention window sizes ( $CW_{min}$ ). After ward, the contention window sizes are decreased from slot by slot during the idle period more than DIFS time. A contending station can send a data frame through wireless channel when the contention window sizes are counted down to zero. If the transmission is unsuccessful or the collisions happen, the contention window size is doubled for every transmission failure until it reaches the maximum contention window sizes ( $CW_{max}$ ). During countdown process in backoff procedure, the contention window will pause if the channel is sensed busy. The backoff countdown process is reactivated when the channel is sensed idle more than DIFS time again. The contention window sizes  $CW_i$  equal  $2^i CW_{min}$ ,  $i = 0, 1, 2, 3 \dots m$ , where  $i$  is the number of backoff stage or the number of retransmissions.  $m$  is the maximum backoff stages, so the maximum contention window size is  $2^m CW_{min}$ . If a destination or a receiver does not receives an acknowledgement frame within an acknowledgement timeout period after a data frame is transmitted, it will continue to retransmit the data frame according to the backoff algorithm. After a successful transmission, the contention window size is reset to the initial value ( $CW_{min}$ ).

### 2.2. Saturation Throughput

The saturated throughput and packet delay are two the most parameters for performances analysis of wireless local area network. In [4], the author has represented the saturation throughput for a finite number of contending stations condition. The saturated throughput can be calculated by dividing the time utilized for transmitting a data packet (payload information) in a slot time by the average duration of a slot time. The throughput equation is given by

$$\text{Throughput of Bianchi's model} = S_{\text{Bianchi's model}} = \frac{E[\text{Payload Information in a slot time}]}{E[\text{Length of a slot time}]} \tag{2}$$

$$S_{[Bianchi's\ model]} = \frac{P_s P_{tr} (MSDU \times 8)}{(1 - P_{tr}) T_{Slot} + P_s P_{tr} T_s + P_c T_c} \quad (3)$$

$$P_{tr} = 1 - (1 - \tau_{[Bianchi's\ model]})^n \quad (4)$$

$$\text{and} \quad P_s = \frac{n \tau_{[Bianchi's\ model]} (1 - \tau_{[Bianchi's\ model]})^{n-1}}{P_{tr}}$$

$$P_s = \frac{n \tau_{[Bianchi's\ model]} (1 - \tau_{[Bianchi's\ model]})^{n-1}}{1 - (1 - \tau_{[Bianchi's\ model]})^n} \quad (5)$$

$$\text{and} \quad P_c = P_{tr} (1 - P_s)$$

Where

$P_{tr}$  = the probability that in a slot time there is at least one transmission

$P_s$  = the successful probability in a slot time

$P_{tr} P_s$  = the probability of successful transmission

$(1 - P_{tr})$  = the probability that a slot time is empty

$P_c = P_{tr} (1 - P_s)$  = the probability of collision transmission

$T_c$  = the collision transmission time in  $\mu s$

$T_s$  = the successful transmission time in  $\mu s$

$MSDU$  = the MAC service data unit size in bytes

### 3. The Fixed Backoff Stages and Fixed Contention Windows technique (FBFC technique)

A new discrete Markov chain model uses fixed backoff stages and fixed contention windows (FBFC) technique. The relation of contention window sizes (CW) and backoff stages ( $i$ ) in BEB algorithm can see in Figure 2. Similarly Bianchi's model, we use binary exponential backoff (BEB) algorithm to solve the collision problem in wireless LAN channels. The contention window sizes in backoff mode are fixed in range minimum contention window sizes (CWmin) to maximum contention window sizes (CWmax). In this model, the CWmin is fixed at 8 timeslots, and the CWmax is fixed at 1024 timeslots. A contention window size depends on the collision probability and backoff stages ( $i$ ) for a data frame transmission. In our model, the state probability of each backoff stages and contention window sizes are denoted by  $b_{i,k}$  where  $i$  indicates the backoff stages, and  $k$  indicates the contention window sizes. The backoff stages  $i$  vary from 0 to 7 stages and the contention window sizes  $k$  vary from 0 to 1023 timeslots. Moreover, we consider the effect of the paused probability in countdown contention windows process. This phenomenon occurs when the channel is sensed busy because the other contending stations start to send a data packet. The countdown contention window process is paused until the channel is idle more than the period of DIFS again. Afterward, the countdown contention window process resumes. The paused probability is denoted  $P_F$ .

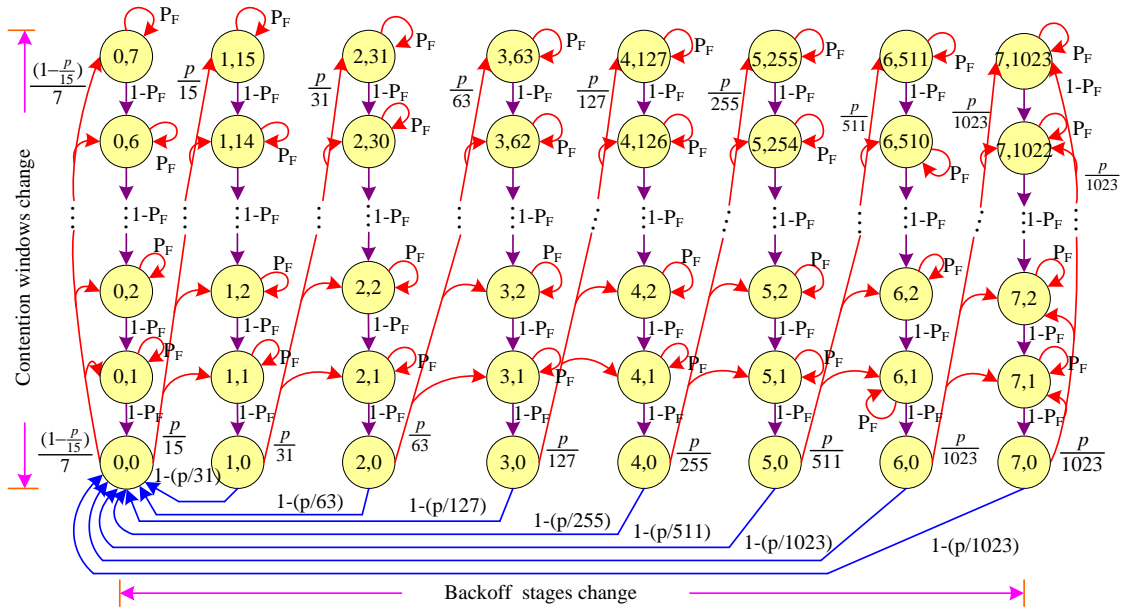


Fig. 2. The Fixed Backoff stages and Fixed Contention windows (FBFC) technique (proposed model)

In figure 2, we use the global balance equation concept to derive the transmission probability. Firstly, we consider in case of backoff stage ( $i$ ) = 0 and contention windows ( $k$ ) = 7 timeslots. The state probability of  $b_{0,7}$  can be described by

$$\begin{aligned} \left(\frac{1-p}{15}\right) b_{0,0} + P_F b_{0,7} &= (1-P_F) b_{0,7} \\ \left(\frac{1-p}{15}\right) b_{0,0} &= (1-2P_F) b_{0,7} \\ b_{0,7} &= \frac{\left(\frac{1-p}{15}\right)}{7(1-2P_F)} b_{0,0} \end{aligned} \tag{6}$$

Next step: in case of backoff stage ( $i$ ) = 0 and contention windows ( $k$ ) = 6 timeslots, the state probability of  $b_{0,6}$  can be described by

$$\begin{aligned} \left(\frac{1-p}{15}\right) b_{0,0} + P_F b_{0,6} + (1-P_F) b_{0,7} &= (1-P_F) b_{0,6} \\ \left(\frac{1-p}{15}\right) b_{0,0} + (1-P_F) b_{0,7} &= (1-2P_F) b_{0,6} \end{aligned}$$

$$b_{0,6} = \frac{\left(1 - \frac{p}{15}\right)}{7(1-2P_F)} b_{0,0} + \frac{(1-P_F)}{(1-2P_F)} b_{0,7} \quad (7)$$

Substituting (6) in to (7), we get

$$\begin{aligned} b_{0,6} &= \frac{\left(1 - \frac{p}{15}\right)}{7(1-2P_F)} b_{0,0} + \frac{(1-P_F)}{7(1-2P_F)} \frac{\left(1 - \frac{p}{15}\right)}{(1-2P_F)} b_{0,0} \\ b_{0,6} &= \frac{\left(1 - \frac{p}{15}\right)}{7(1-2P_F)} b_{0,0} + \frac{\left(1 - \frac{p}{15}\right)}{7(1-P_F)} \frac{(1-P_F)^2}{(1-2P_F)^2} b_{0,0} \\ b_{0,6} &= \frac{\left(1 - \frac{p}{15}\right)}{7(1-P_F)} \left[ \frac{(1-P_F)}{(1-2P_F)} \right] b_{0,0} + \frac{\left(1 - \frac{p}{15}\right)}{7(1-P_F)} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^2 b_{0,0} \\ b_{0,6} &= \frac{\left(1 - \frac{p}{15}\right)}{7(1-P_F)} \sum_{L=1}^2 \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{0,0} \end{aligned} \quad (8)$$

From (6) and (8), at backoff stage ( $i$ ) = 0 and contention windows ( $k$ ) = 1 timeslots. We can summarize that the state probability of  $b_{0,1}$  is given by

$$b_{0,1} = \frac{\left(1 - \frac{p}{15}\right)}{7(1-P_F)} \sum_{L=1}^7 \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{0,0} \quad (9)$$

Next step: in case of  $i = 0$  and  $k = 0$ , the state probability of  $b_{0,0}$  is given by

$$\begin{aligned} \frac{\left(1 - \frac{p}{15}\right)}{7} b_{0,0} + \frac{p}{15} b_{0,0} &= (1-P_F) b_{0,1} + \left(1 - \frac{p}{31}\right) b_{1,0} + \left(1 - \frac{p}{63}\right) b_{2,0} + \left(1 - \frac{p}{127}\right) b_{3,0} + \left(1 - \frac{p}{255}\right) b_{4,0} + \left(1 - \frac{p}{511}\right) b_{5,0} \\ &+ \left(1 - \frac{p}{1023}\right) b_{6,0} + \left(1 - \frac{p}{1023}\right) b_{7,0} \end{aligned} \quad (10)$$

Substituting (9) in to (10) and simplifying the result, we get

$$\begin{aligned} \left[ \frac{15-6p}{15 \times 7} - \frac{(15-p)}{15 \times 7} \sum_{L=1}^7 \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L \right] b_{0,0} &= \left(1 - \frac{p}{31}\right) b_{1,0} + \left(1 - \frac{p}{63}\right) b_{2,0} + \left(1 - \frac{p}{127}\right) b_{3,0} + \left(1 - \frac{p}{255}\right) b_{4,0} + \\ &\left(1 - \frac{p}{511}\right) b_{5,0} + \left(1 - \frac{p}{1023}\right) b_{6,0} + \left(1 - \frac{p}{1023}\right) b_{7,0} \end{aligned}$$

Finally, the state probability of  $b_{0,0}$  is

$$b_{0,0} = A \left[ \begin{aligned} &\left(1 - \frac{p}{31}\right) b_{1,0} + \left(1 - \frac{p}{63}\right) b_{2,0} + \left(1 - \frac{p}{127}\right) b_{3,0} + \left(1 - \frac{p}{255}\right) b_{4,0} \\ &+ \left(1 - \frac{p}{511}\right) b_{5,0} + \left(1 - \frac{p}{1023}\right) b_{6,0} + \left(1 - \frac{p}{1023}\right) b_{7,0} \end{aligned} \right] \quad (11)$$

Where

$$A = \left[ \frac{1}{\left( \frac{(15-6p)}{105} - \frac{(15-p)}{105} \sum_{L=1}^7 \left[ \frac{(1-P_F)}{1-2P_F} \right]^L \right)} \right]$$

Next backoff state: in case of backoff states ( $i$ ) = 1 and contention windows ( $k$ ) = 15 timeslots. The state probability of  $b_{1,15}$  can be described by

$$\begin{aligned} \frac{P}{15} b_{0,0} + P_F b_{1,15} &= (1 - P_F) b_{0,15} \\ (1 - 2P_F) b_{0,15} &= \frac{P}{15} b_{0,0} \\ b_{1,15} &= \frac{P}{15(1 - 2P_F)} b_{0,0} = \frac{P}{15(1 - P_F)} \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right] b_{0,0} \end{aligned} \tag{12}$$

Next step: in case of  $i=1$  and  $k = 14$ , the state probability of  $b_{1,14}$  is given by

$$\begin{aligned} \frac{P}{15} b_{0,0} + P_F b_{1,14} + (1 - P_F) b_{1,15} &= (1 - P_F) b_{1,14} \\ (1 - 2P_F) b_{1,14} &= \frac{P}{15} b_{0,0} + (1 - P_F) b_{1,15} \\ b_{1,14} &= \frac{P}{15(1 - 2P_F)} b_{0,0} + \frac{(1 - P_F)}{(1 - 2P_F)} b_{1,15} \end{aligned} \tag{13}$$

Substituting (12) in to (13), we get

$$\begin{aligned} b_{1,14} &= \frac{P}{15(1 - 2P_F)} b_{0,0} + \frac{(1 - P_F)}{(1 - 2P_F)} \left[ \frac{P}{15(1 - 2P_F)} \right] b_{0,0} \\ b_{1,14} &= \frac{P}{15(1 - 2P_F)} b_{0,0} + \frac{P(1 - P_F)}{15(1 - 2P_F)^2} b_{0,0} \\ b_{1,14} &= \frac{P}{15(1 - P_F)} \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right] b_{0,0} + \frac{P}{15(1 - P_F)} \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right]^2 b_{0,0} \\ b_{1,14} &= \frac{P}{15(1 - P_F)} \sum_{L=1}^2 \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right]^L b_{0,0} \end{aligned} \tag{14}$$

Next step: in case of  $i=1$  and  $k = 13$ , the state probability of  $b_{1,13}$  is given by

$$\frac{P}{15} b_{0,0} + P_F b_{1,13} + (1 - P_F) b_{1,14} = (1 - P_F) b_{0,13}$$

$$\begin{aligned}
(1-2P_F)b_{1,13} &= \frac{p}{15}b_{0,0} + (1-P_F)b_{1,14} \\
b_{1,13} &= \frac{p}{15(1-2P_F)}b_{0,0} + \frac{(1-P_F)}{(1-2P_F)}b_{1,14}
\end{aligned} \tag{15}$$

Substituting (14) in to (15), we get

$$\begin{aligned}
b_{1,13} &= \frac{p}{15(1-2P_F)}b_{0,0} + \frac{(1-P_F)}{(1-2P_F)} \left[ \frac{p}{15(1-P_F)} \left[ \frac{(1-P_F)}{(1-2P_F)} \right] b_{0,0} + \frac{p}{15(1-P_F)} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^2 b_{0,0} \right] \\
b_{1,13} &= \frac{p}{15(1-P_F)} \left[ \frac{(1-P_F)}{(1-2P_F)} \right] b_{0,0} + \frac{p}{15(1-P_F)} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^2 b_{0,0} + \frac{p}{15(1-P_F)} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^3 b_{0,0} \\
b_{1,13} &= \frac{p}{15(1-P_F)} \sum_{L=1}^3 \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{0,0}
\end{aligned} \tag{16}$$

From (12) (14) and (16), we can summarize that the state probability of  $b_{1,1}$  is given by

$$b_{1,1} = \frac{p}{15(1-P_F)} \sum_{L=1}^{15} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{0,0} \tag{17}$$

Next step: in case of  $i=1$  and  $k=0$ , the state probability of  $b_{1,0}$  is expressed as

$$\begin{aligned}
\left(1 - \frac{p}{31}\right)b_{1,0} + \frac{p}{31}b_{1,0} &= (1-P_F)b_{1,1} \\
\left(1 - \frac{p}{31} + \frac{p}{31}\right)b_{1,0} &= (1-P_F)b_{1,1} \\
b_{1,0} &= (1-P_F)b_{1,1}
\end{aligned} \tag{18}$$

Substituting (17) in to (18), the state probability of  $b_{1,0}$  can be calculated by

$$\begin{aligned}
b_{1,0} &= (1-P_F) \left[ \frac{p}{15(1-P_F)} \sum_{L=1}^{15} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{0,0} \right] \\
b_{1,0} &= \frac{p}{15} \sum_{L=1}^{15} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{0,0}
\end{aligned} \tag{19}$$

Next backoff state: in case of backoff state ( $i$ ) = 2 and contention windows ( $k$ ) = 31 timeslots, the state probability of  $b_{2,31}$  can be described by

$$\frac{p}{31}b_{1,0} + P_F b_{2,31} = (1-P_F)b_{2,31}$$



$$(1 - 2P_F) b_{2,31} = \frac{P}{31} b_{1,0}$$

$$b_{2,31} = \frac{P}{31(1 - 2P_F)} b_{1,0} \tag{20}$$

Next step: in case of  $i = 2$  and  $k = 30$ , the state probability of  $b_{2,30}$  is given by

$$\frac{P}{31} b_{1,0} + P_F b_{2,30} + (1 - P_F) b_{2,31} = (1 - P_F) b_{2,30}$$

$$(1 - 2P_F) b_{2,30} = \frac{P}{31} b_{1,0} + (1 - P_F) b_{2,31}$$

$$b_{2,30} = \frac{P}{31(1 - 2P_F)} b_{1,0} + \frac{(1 - P_F)}{(1 - 2P_F)} b_{2,31} \tag{21}$$

Substituting (20) in to (21), we get

$$b_{2,30} = \frac{P}{31(1 - 2P_F)} b_{1,0} + \frac{(1 - P_F)}{(1 - 2P_F)} \left[ \frac{P}{31(1 - 2P_F)} \right] b_{1,0}$$

$$b_{2,30} = \frac{P}{31(1 - 2P_F)} b_{1,0} + \frac{P(1 - P_F)}{31(1 - 2P_F)^2} b_{1,0}$$

$$b_{2,30} = \frac{P}{31(1 - P_F)} \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right] b_{1,0} + \frac{P}{31(1 - P_F)} \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right]^2 b_{1,0} = \frac{P}{31(1 - P_F)} \sum_{L=1}^2 \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right]^L b_{1,0} \tag{22}$$

From (20) and (22), we can summarize that the state probability at  $i = 2$  and  $k = 1$  is given by

$$b_{2,1} = \frac{P}{31(1 - P_F)} \sum_{L=1}^{31} \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right]^L b_{1,0} \tag{23}$$

In case of  $i = 2$  and  $k = 0$ , the state probability of  $b_{2,0}$  can be expressed by

$$\left( 1 - \frac{P}{63} \right) b_{2,0} + \frac{P}{63} b_{2,0} = (1 - P_F) b_{2,1}$$

$$\left( 1 - \frac{P}{63} + \frac{P}{63} \right) b_{2,0} = (1 - P_F) b_{2,1}$$

$$b_{2,0} = (1 - P_F) b_{2,1} \tag{24}$$

Substituting (23) in to (24), we get

$$b_{2,0} = (1 - P_F) \left[ \frac{P}{31(1 - P_F)} \sum_{L=1}^{31} \left[ \frac{(1 - P_F)}{(1 - 2P_F)} \right]^L b_{1,0} \right]$$

$$b_{2,0} = \frac{p}{31(1-P_F)} \sum_{L=1}^{31} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{1,0} \quad (25)$$

From (19) and (25), we can summarize that state probability at  $i = 3$  and  $k = 0$  is given by

$$b_{3,0} = \frac{p}{63} \sum_{L=1}^{63} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{2,0} \quad (26)$$

Similarly: in case of  $i = 4$  and  $k = 0$ , the state probability of  $b_{4,0}$  is given by

$$b_{4,0} = \frac{p}{127} \sum_{L=1}^{127} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{3,0} \quad (27)$$

Similarly: in case of  $i = 5$  and  $k = 0$ , the state probability of  $b_{5,0}$  is given by

$$b_{5,0} = \frac{p}{255} \sum_{L=1}^{255} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{4,0} \quad (28)$$

Similarly: in case of  $i = 6$  and  $k = 0$ , the state probability of  $b_{6,0}$  is given by

$$b_{6,0} = \frac{p}{511} \sum_{L=1}^{511} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{5,0} \quad (29)$$

Next backoff state: in case of  $i = 7$  and  $k = 1023$ , the state probability of  $b_{7,1023}$  is given by

$$\begin{aligned} \frac{p}{1023} b_{6,0} + \frac{p}{1023} b_{7,0} + P_F b_{7,1023} &= (1-P_F) b_{7,1023} \\ (1-2P_F) b_{7,1023} &= \frac{p}{1023} b_{6,0} + \frac{p}{1023} b_{7,0} \\ b_{7,1023} &= \frac{p}{1023(1-2P_F)} b_{6,0} + \frac{p}{1023(1-2P_F)} b_{7,0} \end{aligned} \quad (30)$$

In case of  $i = 7$  and  $k = 1022$ , the state probability of  $b_{7,1022}$  is given by

$$\begin{aligned} \frac{p}{1023} b_{6,0} + \frac{p}{1023} b_{7,0} + (1-P_F) b_{7,1023} + P_F b_{7,1022} &= (1-P_F) b_{7,1022} \\ (1-2P_F) b_{7,1022} &= \frac{p}{1023} b_{6,0} + \frac{p}{1023} b_{7,0} + (1-P_F) b_{7,1023} \\ b_{7,1022} &= \frac{p}{1023(1-2P_F)} b_{6,0} + \frac{p}{1023(1-2P_F)} b_{7,0} + \frac{(1-P_F)}{(1-2P_F)} b_{7,1023} \end{aligned} \quad (31)$$

Substituting (30) in to (31), we get

$$\begin{aligned} b_{7,1022} &= \frac{p}{1023(1-2P_F)} b_{6,0} + \frac{p}{1023(1-P_F)} b_{7,0} + \frac{(1-P_F)}{(1-2P_F)} \left[ \frac{p}{1023(1-2P_F)} b_{6,0} + \frac{p}{1023(1-2P_F)} b_{7,0} \right] \\ b_{7,1022} &= \frac{p}{1023(1-2P_F)} b_{6,0} + \frac{p(1-P_F)}{1023(1-P_F)^2} b_{6,0} + \frac{p}{1023(1-P_F)} b_{7,0} + \frac{p(1-P_F)}{1023(1-2P_F)^2} b_{7,0} \end{aligned}$$

$$b_{7,1022} = \frac{p}{1023(1-P_F)} \sum_{L=1}^2 \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{6,0} + \frac{p}{1023(1-P_F)} \sum_{L=1}^2 \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{7,0} \tag{32}$$

From (30) and (32), we can summarize that the state probability of  $b_{7,1}$  is given by

$$b_{7,1} = \frac{p}{1023(1-P_F)} \sum_{L=1}^{1023} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{6,0} + \frac{p}{1023(1-P_F)} \sum_{L=1}^{1023} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{7,0} \tag{33}$$

Next step: in case of  $i = 7$  and  $k = 0$ , the stage probability of  $b_{7,0}$  can be expressed by

$$\begin{aligned} \left(1 - \frac{p}{1023}\right) b_{7,0} + \frac{p}{1023} b_{7,0} &= (1 - P_F) b_{7,1} \\ \left(1 - \frac{p}{1023} + \frac{p}{1023}\right) b_{7,0} &= (1 - P_F) b_{7,1} \\ b_{7,0} &= (1 - P_F) b_{7,1} \end{aligned} \tag{34}$$

Substituting (33) in to (34), we get

$$b_{7,0} = \frac{p}{1023} \sum_{L=1}^{1023} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{6,0} + \frac{p}{1023} \sum_{L=1}^{1023} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L b_{7,0} \tag{35}$$

From (19) (25) (26) (27) (28) (29) and (35), we assign

$$\begin{aligned} B &= \frac{p}{15} \sum_{L=1}^{15} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L, & C &= \frac{p}{31} \sum_{L=1}^{31} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L \\ D &= \frac{p}{63} \sum_{L=1}^{63} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L, & E &= \frac{p}{127} \sum_{L=1}^{127} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L \\ F &= \frac{p}{255} \sum_{L=1}^{255} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L, & G &= \frac{p}{511} \sum_{L=1}^{511} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L \\ H &= \frac{p}{1023} \sum_{L=1}^{1023} \left[ \frac{(1-P_F)}{(1-2P_F)} \right]^L \end{aligned}$$

Therefore, the state probability of (19) (25) (26) (27) (28) (29) and (35) can be rewritten by

$$b_{1,0} = Bb_{0,0} \tag{36}$$

$$b_{2,0} = Cb_{1,0} = BCb_{0,0} \tag{37}$$

$$b_{3,0} = Db_{2,0} = BCDB_{0,0} \tag{38}$$

$$b_{4,0} = Eb_{3,0} = BCDEb_{0,0} \tag{39}$$

$$b_{5,0} = Fb_{4,0} = BCDEFb_{0,0} \quad (40)$$

$$b_{6,0} = Gb_{5,0} = BCDEFGb_{0,0} \quad (41)$$

$$b_{7,0} = Hb_{6,0} + Hb_{7,0} \quad (42)$$

$$(1 - H)b_{7,0} = Hb_{6,0}$$

$$b_{7,0} = \frac{H}{(1 - H)} b_{6,0} \quad (43)$$

The equations (36) to (43) describe the backoff state probability ( $b_{i,k}$ ) which is the function of the collision probability ( $p$ ), the paused probability ( $p_f$ ) and the contention window sizes ( $CW$ ). The transmission probability ( $\tau$ ) occurs when the contention windows are decreased to zero. In saturation case, the summation of state probability ( $b_{i,0}$ ) equals one. Finally, the new transmission probability ( $\tau_{[proposed\ model]}$ ) of Binary Exponential Backoff algorithm in FBFC model is

$$\tau_{[proposed\ model]} = \sum_{i=0}^7 b_{i,0} = b_{0,0} + b_{1,0} + b_{2,0} + b_{3,0} + b_{4,0} + b_{5,0} + b_{6,0} + b_{7,0} = 1 \quad (44)$$

Substituting (36) (37) (38) (39) (40) (41) and (43) in to (44), we get

$$\tau_{[proposed\ model]} = b_{0,0} + Bb_{0,0} + BCb_{0,0} + BCDb_{0,0} + BCDEb_{0,0} + BCDEFb_{0,0} + BCDEFGb_{0,0} + BCDEFG\left(\frac{H}{1-H}\right)b_{0,0} \quad (45)$$

$$\tau_{[proposed\ model]} = \left[ 1 + B + BC + BCD + BCDE + BCDEF + BCDEFG + BCDEFG \frac{H}{(1-H)} \right] b_{0,0} = 1 \quad (46)$$

Finally, the transmission probability of Binary Exponential Backoff algorithm is given by

$$\tau_{[FBFC\ model]} = \frac{1}{\left[ 1 + B + BC + BCD + BCDE + BCDEF + BCDEFG + \left[ \frac{H}{1-H} \right] BCDEFG \right]} \quad (47)$$

Equation (47) is the transmission probability of a new discrete Markov chain model by using the fixed backoff stages and fixed contention window sizes technique.

#### 4. Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) Protocol

In wireless local area network channel, the medium access technique uses the CSMA/CA protocol in distribute coordination function (DCF) mode for IEEE802.11a/b/g standards. In CSMA/CA technique, there are two access methods: the first is Basic Access method (BA), and the second is Request-to-Send and Clear-to-Send (RTS/CTS) access methods. The RTS/CTS technique has been introduced to reduce the performance degradation due to hidden terminal, and this paper considers only the CSMA/CA protocol in RTS/CTS technique. A data frame exchange sequence of CSMA/CA with RTS CTS mechanism in IEEE802.11a/b/g standards is shown in Fig. 3. Before transmitting a data frame MSDU (MAC service data unit), a shot RTS frame is transmitted. If the RTS frame success, the receiver station responds with a shot CTS frame. Then, a data frame and an ACK frame will follow. All four frames (RTS, CTS, DATA and ACK) are separated by SIFS time.

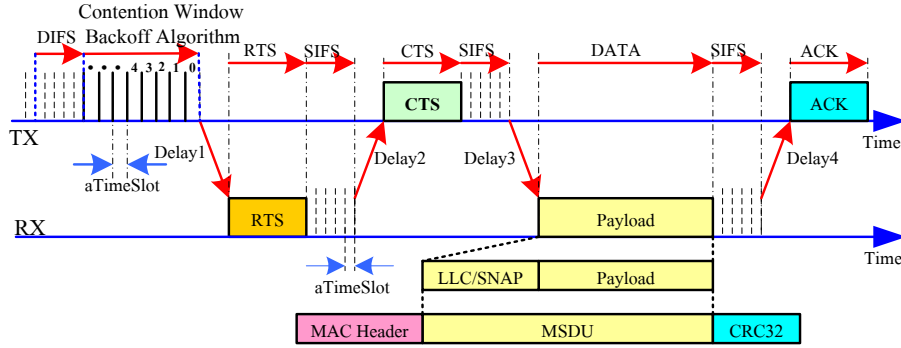


Fig. 3. A data transmission procedure of CSMA/CA with RTS CTS Protocol

Where

- DIFS = Distributed coordination function Inter Frame Space
- SIFS = Short Inter Frame Space
- RTS = Request-to-Send frame
- CTS = Clear-to-Send frame
- ACK = Acknowledgement frame
- MSDU = MAC Service Data Unit frame

The time periods of  $T_s$  and  $T_c$  for CSMA/CA with RTS CTS protocol are obtained as follows

$$T_{S[CSMA/CA\ RTS\ CTS]} = T_{RTS} + 3T_{SIFS} + 4T_{delay} + T_{CTS} + T_{MSDU(size)} + T_{ACK} + T_{DIFS} \quad (48)$$

$$T_{C[CSMA/CA\ RTS\ CTS]} = T_{DIFS} + T_{RTS} + T_{delay} \quad (49)$$

$$T_{MSDU} = \frac{MSDU \times 8}{Data\ rate} \quad (50)$$

From (2) (3) (4) and (5), a new saturated throughput equation of WLAN system in FBFC scheme can be rewritten as

$$S_{[FBFC\ model]} = \frac{P_{S[FBFC\ model]} P_{tr[FBFC\ model]} (MSDU \times 8)}{(1 - P_{tr[FBFC\ model]}) T_{Slot} + P_{S[FBFC\ model]} P_{tr[FBFC\ model]} T_{S[FBFC\ model]} + P_{C[FBFC\ model]} T_C} \quad (51)$$

$$P_{tr[FBFC\ model]} = 1 - (1 - \tau_{[FBFC\ model]})^n \quad (52)$$

$$and\ P_{S[FBFC\ model]} = \frac{n \tau_{[FBFC\ model]} (1 - \tau_{[FBFC\ model]})^{n-1}}{P_{tr[FBFC\ model]}}$$

$$P_{S[FBFC\ model]} = \frac{n \tau_{[FBFC\ model]} (1 - \tau_{[FBFC\ model]})^{n-1}}{1 - (1 - \tau_{[FBFC\ model]})^n} \quad (53)$$

$$and\ P_{C[FBFC\ model]} = P_{tr[FBFC\ model]} (1 - P_{S[FBFC\ model]}) \quad (54)$$

The comparison of throughput efficiency between Bianchi’s model and FBFC model use the same Physical layer parameters which listed in Table 1. The physical layer of IEEE802.11b is the direct sequence spread spectrum (DSSS), and the physical layer of IEEE802.11a and IEEE802.11g are the orthogonal frequency division

multiplexing (OFDM). MathCAD engineering tool in [6] uses for calculation the saturation throughput efficiency. The throughput calculation procedure is given by

Begin

Step 1: Fixed parameters  $p := 0.05$ ,  $P_f := 0.05$ ,  $m := 6$ , MSDU: = 1024 and  $n := 1..40$

$CW := 31, 63, 127, 255, 511$  and 1023

Step 2: calculated  $T_{S[CSMA/CA RTS CTS]}$  by used equation (48) and (50)

Step 3: calculated  $T_{C[CSMA/CA RTS CTS]}$  by used equation (49)

Step 4: calculated  $\tau_{[Bianchi's\ model]}$  by used equation (1)

Step 5: calculated  $P_{tr}$  of Bianchi's model by used equation (4)

Step 6: calculated  $P_s$  of Bianchi's model by used equation (5)

Step 7: calculated the saturated throughput of Bianchi's model by used equation (3)

Step 8: calculated  $\tau_{[FBFC\ model]}$  by used equation (47)

Step 9: calculated  $P_{tr}$  of the Fixed Backoff stages and Fixed contention windows model

(FBFC model) by used equation (52)

Step 10: calculated  $P_s$  of the Fixed Backoff stages and Fixed contention windows model

(FBFC model) by used equation (53)

Step 11: calculated the saturated throughput of proposed model by used equation (51)

End

Table 1. The transmission times in CSMA/CA RTS CTS Protocol [1], [2], [3], [6] and [7]

Transmissions description	IEEE802.11a	IEEE802.11b	IEEE802.11g
$T_{SIFS}$	16 $\mu$ s	10 $\mu$ s	10 $\mu$ s
$T_{DIFS}$	34 $\mu$ s	50 $\mu$ s	28 $\mu$ s
$T_{aSlotTime}$	9 $\mu$ s	20 $\mu$ s	9 $\mu$ s
$T_{delay}$	1 $\mu$ s	1 $\mu$ s	1 $\mu$ s
$T_{RTS}$ OFDM 24-Mbps	28 $\mu$ s	-	34 $\mu$ s
$T_{CTS}$ OFDM 24-Mbps	28 $\mu$ s	-	32 $\mu$ s
$T_{ACK}$ OFDM 24-Mbps	28 $\mu$ s	-	32 $\mu$ s
$T_{RTS}$ OFDM 54-Mbps	24 $\mu$ s	-	30 $\mu$ s
$T_{CTS}$ OFDM 54-Mbps	24 $\mu$ s	-	30 $\mu$ s
$T_{ACK}$ OFDM 54-Mbps	24 $\mu$ s	-	30 $\mu$ s
$T_{RTS}$ HR 11-Mbps	-	352 $\mu$ s	-
$T_{CTS}$ HR 11-Mbps	-	304 $\mu$ s	-
$T_{ACK}$ HR 11-Mbps	-	304 $\mu$ s	-
CWmin	15 SlotTimes	31 SlotTimes	16 SlotTimes
CWmax	1023 SlotTimes	1023 SlotTimes	1024 SlotTimes

### 5. Numerical Results

In this section, the numerical results show the saturated throughput of a new discrete Markov chain models (FBFC technique) and the saturated throughput of a legacy model (Bianchi’s model). In our computer simulations, the physical layer parameters in saturated throughput equations use the same parameters as [6] and [7]. The important parameter of this research is the transmission probability ( $\tau$ ) that is derived from a new discrete Markov chain model ( $\tau_{[FBFC\ model]}$ ) and Bianchi’s model ( $\tau_{[Bianchi\ model]}$ ). The transmission probability is the key for the comparison saturation throughput performance between Bianchi’s model and FBFC model. In both model, the contention windows vary from 15 to 1024 timeslots, and the numbers of contending stations vary from 1 to 40 stations, and the backoff states vary from 0 to 7 stages. Also, the collision probability ( $p$ ), the paused station probability ( $P_F$ ), and the MAC service data unit sizes are fixed at 0.05, 0.05 and 1024 bytes, respectively. First of all, figure 4 shows the comparison saturated throughput by using the physical layer parameters in IEEE802.11b standard which based on direct sequence spread spectrum (DSSS).

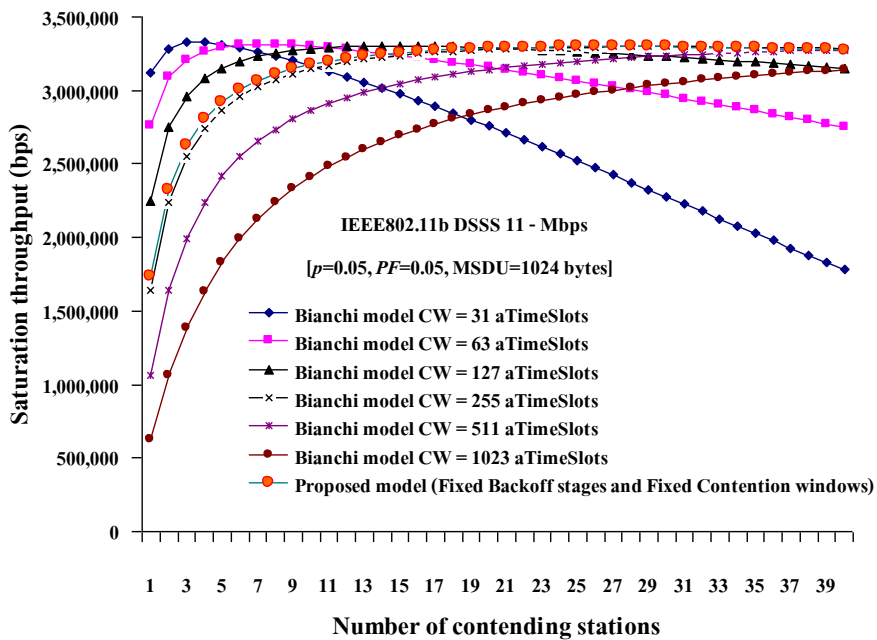


Fig. 4. Throughput performance of the Fixed Backoff stages and Fixed Contention windows technique in IEEE802.11b standard

From the results in Fig.4, when the contention windows are fixed at 31, 63 and 127 timeslots, and the contending stations are varied from 1 to 20 stations, the saturated throughput of the legacy model (Bianchi’s model) is better than the FBFC model (proposed model). However, when the contending stations are fixed at 255, 511 and 1023 timeslots, the throughput performance of FBFC model is higher than the Bianchi’s model. Afterward, when the contending stations are varied from 21 to 40 stations, the saturated throughput of FBFC model is stable and equal as the Bianchi’s model. Surprisingly, when the contention windows are 31 and 63 aTimeSlots, the throughput of Bianchi’s model seems to reduce quickly, but the saturated throughput of FBFC model seems to be stable. Therefore, from the results, we can summarize that the FCFB model is suitable than the Bianchi’s model in heavy traffic load condition.

Afterward, the figure 5 and 6 show the comparison throughput efficiency between FBFC model and Bianchi’s model in IEEE802.11a and IEEE802.11g standards. The physical layers are based on the orthogonal

frequency division multiplexing (OFDM), where the data speed of 802.11a is fixed at 24 Mbps and data speed of 802.11g is fixed at 54 Mbps. Similarly, when the contending stations are varied from 1 to 22 stations, and the contention windows are fixed at 31, 63 and 127 timeslots, the throughput efficiency of Bianchi's model is higher than the FBFC model. On the contrary, when the contention windows are fixed at 255, 511 and 1023 timeslots, the saturated throughput efficiency of FBFC model is higher than the Bianchi's model. From the saturated throughput in Fig. 4, Fig.5 and Fig.6, we can summarize that the transmission probability of proposed model ( $\tau_{[FBFC\ model]}$ ) is the average of Bianchi's model. Furthermore, we also conclude that the new discrete Markov chain model is derived from the fixed backoff stages and fixed contention windows technique have a good throughput at high contending stations condition.

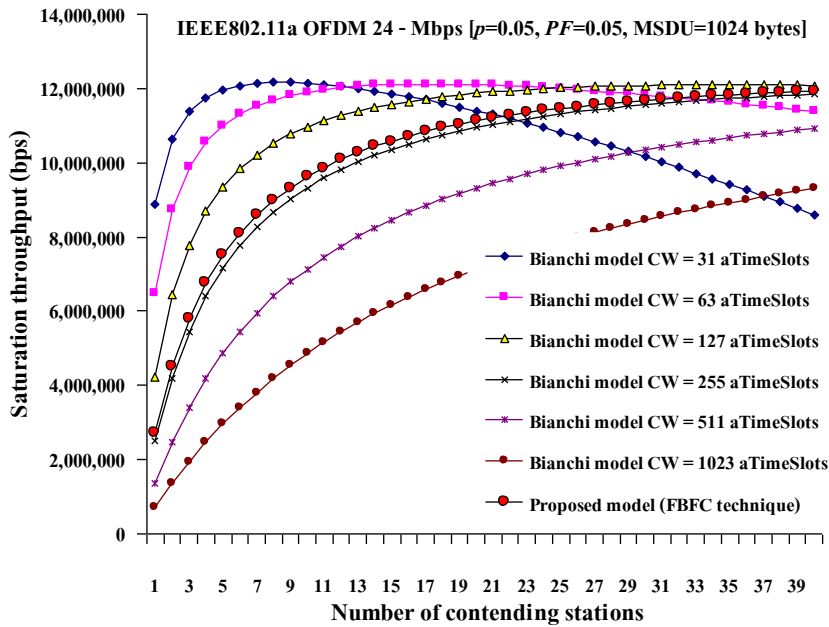


Fig. 5. Throughput performance of the Fixed Backoff stages and Fixed Contention windows technique in IEEE802.11a standard



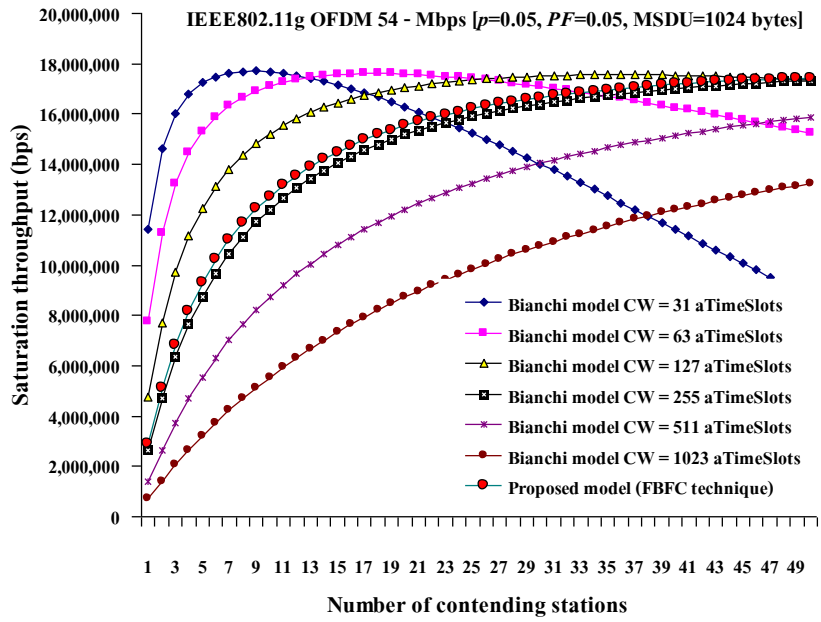


Fig. 6. Throughput performance of the Fixed Backoff stages and Fixed Contention windows technique in IEEE802.11g standard

Obviously, the results in Fig.5 and Fig. 6 show that when the contending stations are increased, the saturated throughput of Bianchi’s model is reduced at less contention window sizes. However, the saturated throughput of FBFC model is increased up to the stable point.

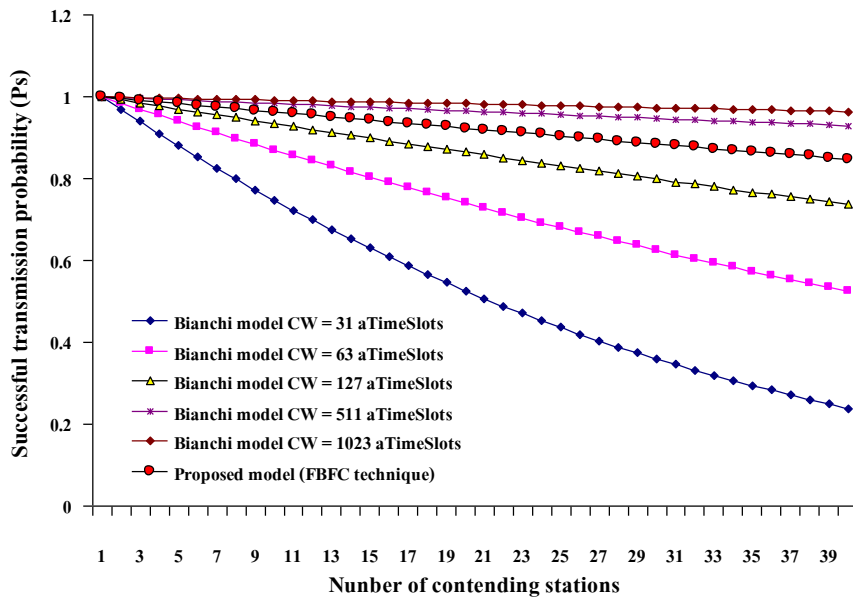


Fig. 7. The successful transmission probability of the Fixed Backoff stages and Fixed Contention windows technique

Figure 7 shows the effect of the number of contending stations on successful transmission probability. From the figure, the successful probability of Bianchi’s model gets higher than Fixed Backoff stages and Fixed Contention

windows model when the contention windows are set at 511 and 1023 timeslots. Dramatically, when the contention windows are set at 31, 63 and 127 timeslots, the successful transmission probability of Bianchi's model seems to reduce quickly, but the successful transmission probability of FBFC model seems to reduce little. In addition, when the contention windows are set at 31 to 1023 timeslots, the simulation results indicate that the successful transmission probability ( $P_s$ ) of FBFC model is the average of Bianchi's model.

## 6. Conclusion

In this research, we introduce a new discrete Markov chain model to calculate the transmission probability in distributed coordination function for wireless local area network system, and it's called the Fixed Backoff stages and Fixed Contention windows technique. The transmission probability parameters of FBFC model ( $\tau_{[FBFC\ model]}$ ) and Bianchi's model ( $\tau_{[Bianchi's\ model]}$ ) in saturated throughput equations are two important parameters for the throughput performance comparison in DCF IEEE802.11a/b/g WLAN standards. Moreover, the transmission probability of FBFC model which is derived from fixed backoff stages and fixed contention windows scheme is the average point of Bianchi's model under the same contention windows range (minimum contention windows to maximum contention windows rang). Our numerical results show that the performance of proposed model (FBFC model) is stable under a lot of contending stations condition, and all numerical results guarantee that the performance of FBFC model well operates under saturated coordination function for wireless local area network system. In future work, we will evaluate the performance of FBFC model under non-saturation WLAN channel, and we will search for a new backoff algorithm which will have the performance better than the legacy backoff scheme under the fixed backoff stages and fixed contention windows concept.

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