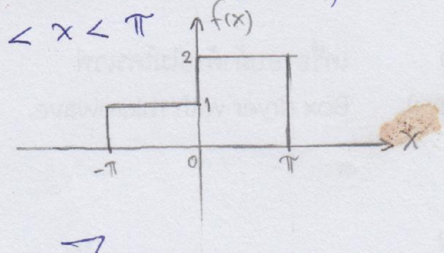
FOURIER SERIES

Ex จงหาอนุกรมฟูเรียร์ของฟังก์ชันซึ่งนิยามโดย

$$f(x) = \begin{cases} 1 & ; \ -\pi < x < 0 \\ 2 & ; \ 0 < x < \pi \end{cases}$$

เมื่อ $f(x+2\pi) = f(x)$ แสดงว่ามีค่า $= 2\pi$



Sol^n

■ หาสัมประสิทธิ์ a_0 จะได้

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 1\,dx + \int_{0}^{\pi} 2\,dx\right]$$

$$= \frac{1}{\pi}\left[x \Big|_{-\pi}^{0} + 2x \Big|_{0}^{\pi} \right] = \frac{1}{\pi}\left[0 - (-\pi) + 2\pi - 2(0)\right]$$

$$= \frac{3\pi}{\pi} = 3 \text{ ———\#\#\#}$$

■ หาสัมประสิทธิ์ a_n จะได้

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 1\cos nx\,dx + \int_{0}^{\pi} 2\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\sin nx \Big|_{-\pi}^{0} + \frac{2}{n}\sin nx \Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n}\left(\cancel{\sin n(0)}^{\,0} - \cancel{\sin n(\pi)}^{\,0}\right) + \frac{2}{n}\left(\cancel{\sin n\pi}^{\,0} - \cancel{\sin n(0)}^{\,0}\right)\right]$$

$$= 0 \quad ; \ n = 1, 2, 3, \ldots$$

FOURIER SERIES

■ หาสัมประสิทธิ์ b_n จะได้

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} 1\sin nx\,dx + \int_{0}^{\pi} 2\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-\frac{1}{n}\cos nx\Big|_{-\pi}^{0} + \frac{-2}{n}\cos nx\Big|_{0}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[-\frac{1}{n}\cos \cancelto{1}{n(0)} + \frac{1}{n}\cos n(-\pi) - \frac{2}{n}\cos n(\pi) + \frac{2}{n}\cos \cancelto{1}{n(0)}\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n} + \frac{1}{n}\left(\cos n\pi - 2\cos n\pi\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n} + \frac{-1}{n}\cos n\pi\right] = \frac{1}{n\pi}\left(1 - \cos n\pi\right)$$

$$b_n = \frac{1 - \cos n\pi}{n\pi}$$

พิจารณาที่ $n = 1, 2, 3, \ldots$ จะได้ว่า $\cos n\pi$

$\cos n\pi = (-1)^n$ จะได้ $b_n = \dfrac{1 - (-1)^n}{n\pi}$, $n = 1, 2, 3, \ldots$

ดังนั้น อนุกรมฟูเรียร์ของฟังก์ชัน $f(x)$ จะได้จากสูตร

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$ แทนค่า จะได้

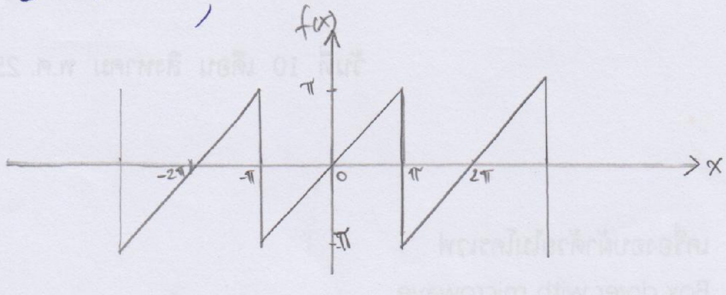
$$= \frac{3}{2} + \sum_{n=1}^{\infty}\frac{1-(-1)^n}{n\pi}\sin nx$$

$$= \frac{3}{2} + \frac{2}{\pi}\left\{\frac{\sin x}{1} + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \cdots\right\}$$ # # #

Fourier Series

Ex จงหาอนุกรมฟูเรียร์ของฟังก์ชันซึ่งนิยามโดย

$$f(x) = x \quad ; \quad -\pi < x < \pi \quad \text{และ} \quad f(x+2\pi) = f(x)$$



Sol ฟังก์ชัน $f(x)$ มีคาบเท่ากับ 2π

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos nx + b_n \sin nx \right)$$

เป็นอนุกรมฟูเรียร์ของฟังก์ชัน $f(x)$ โดย

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx \quad ; \quad n = 1, 2, 3, \dots$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx \quad ; \quad n = 1, 2, 3, \dots$$

หาสัมประสิทธิ์ a_0 จะได้

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{\pi} \left[\int_{-\pi}^{0} -\pi \, dx + \int_{0}^{\pi} \pi \, dx \right]$$

$$= \frac{1}{\pi} \left[-\pi x \Big|_{-\pi}^{0} + \pi x \Big|_{0}^{\pi} \right]$$

$$= \frac{1}{\pi} \left[+\pi(-\pi) + \pi(\pi) \right] = 0 \qquad \#\#\#$$

FOURIER SERIES

หาสัมประสิทธิ์ a_n จะได้

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \;=\; \frac{1}{\pi}\int_{-\pi}^{\pi} x\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\frac{x}{n}\sin nx\Big|_{-\pi}^{\pi} + \frac{1}{n^2}\cos nx\Big|_{-\pi}^{\pi}\right]$$

u	dv
x (+)	$\cos nx$
1 (−)	$\frac{1}{n}\sin nx$
0	$-\frac{1}{n^2}\cos nx$

$$= \frac{1}{\pi}\left[\frac{x}{n}\left(\sin n\pi - \sin n(-\pi)\right) + \frac{1}{n^2}\left(\cos n\pi - \cos n(-\pi)\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{x}{n}\left(\cancelto{0}{\sin n\pi + \sin n\pi}\right) + \frac{1}{n^2}\left(\cancelto{0}{\cos n\pi - \cos n\pi}\right)\right]$$

$$= \frac{1}{\pi}\left(0 + 0\right) = 0 \quad \#\#\#$$

หาสัมประสิทธิ์ b_n จะได้

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \;=\; \frac{1}{\pi}\int_{-\pi}^{\pi} x\sin nx\,dx$$

u	dv
x (+)	$\sin nx$
1 (−)	$-\frac{1}{n}\cos nx$
0	$-\frac{1}{n^2}\sin nx$

$$b_n = \frac{1}{\pi}\left[\frac{-x}{n}\cos nx\Big|_{-\pi}^{\pi} + \frac{1}{n^2}\sin nx\Big|_{-\pi}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{n}\left(\cos n\pi + \frac{(-\pi)}{n}\cos n(-\pi)\right) + \frac{1}{n^2}\left(\sin n\pi - \sin n(-\pi)\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{-\pi}{n}\left(\cos n\pi + \cos n\pi\right) + \frac{1}{n^2}\left(\cancelto{0}{\sin n\pi + \sin n\pi}\right)\right]$$

$$= \frac{1}{\pi}\left(\frac{-\pi}{n}\,2\cos n\pi\right)$$

$$= -\frac{2}{n}\cos n\pi \quad \text{——} \#\#\#$$

FOURIER SERIES

$$\therefore b_n = \frac{-2\cos n\pi}{n}$$

พิจารณา $\cos n\pi$ เมื่อ $n = 1, 2, 3, \ldots$

$-1, 1, -1, 1$ สลับกัน

$$b_n = \frac{2(-1)^{n+1}}{n}$$

$\Rightarrow$ จากสูตรอนุกรมฟูเรียร์ของฟังก์ชัน $f(x)$ คือ

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

$$= \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin nx$$

$$= 2\left\{ \sin x - \frac{\sin 2x}{2} + \frac{\sin 3x}{3} - \frac{\sin 4x}{4} + \ldots \right\} \quad \#\#\#$$